A Fully Four Dimensional Reconstruction Strategy for Cardiac Gated SPECT with Noise Reduction, Scatter Correction, Resolution Restoration and Inversion of Attenuated Radon Transform in KL Space

Yi Fan, Hongbing Lu, and Zhengrong Liang

***Abstract*— Based on our previous work, an efficient, analytical solution to reconstruction problem of cardiac gated SPECT has been developed that allows simultaneous compensation for non-uniform attenuation, scatter, and system-dependent resolution variation, as well as suppression of signal-dependent Poisson noise. Karhune-Loève (KL) transform is a useful tool for the reduction of intensive computational cost by simplifying the 4D reconstruction problem into a series of three dimensional ones. We proved that the noise property of SPECT which follows Poisson distribution remains same in KL domain and so does the transmitted system model. To minimize the errors and blurring caused by global KL transform, we proposed an adaptive grouping method to group frames along time axis into several phases to perform specific KL transform. In the KL domain, non-stationary Poisson noise was stabilized by Anscombe transform and treated by adaptive Wiener filtration. Scatter contribution to the primary energy window was then estimated and removed based on photon detection energy spectrum and the triple-energy-window acquisition formula after noise treatment. The scatter-corrected data was further subject to a depth-dependent deconvolution, based on the distance frequency relationship, with measured detector response kernel in the KL domain. The de-convoluted sinograms were reconstructed by inverting the attenuated Radon transform for each KL component and the 4D SPECT images were obtained by a corresponding inverse KL transform. The simultaneous compensation strategy in the KL domain was tested by computer simulations from digital phantoms of 128 cubic array and clinical data from a patient. The adaptive KL transform for different groups consisting of frames with similar activity dynamics showed noticeable improvement over our previous work of using a single KL transform for all frames. Improvement was also seen by the adaptive noise treatment of all the KL components over previous work of discarding the higher-order components. Further improvement by considering the scatter and resolution variation was demonstrated.**

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Yi Fan was with the Department of Computer application, Fourth Military Medical University Xi'an, Shaanxi 710032, China and now is with the Department of Radiology, State University of New York, Stony Brook, NY 11794 USA (e-mail: yifan@mil.sunysb.edu).

Hongbing Lu is with the Department of Computer application, Fourth Military Medical University, Xi'an, Shaanxi 710032, China (luhb@fmmu.edu.cn)

Zhengrong Liang is with the Department of Radiology, State University of New York, Stony Brook, NY 11794 USA (Jerome.Liang@sunysb.edu).

***Index Terms* — Gated SPECT, image Reconstruction, Karhune-Loève (KL) transform, Noise Reduction**

I. INTRODUCTION

In the study of single photon emission computed tomography (SPECT), gated imaging can yield valuable kinetic information and thus are widely used in clinical treatment. While, due to the shorter time of photon collection for each frame compared with static image, the number of counts per image is lowered and thus the data in each frame are much more polluted by noise. To achieve quantitative reconstruction for clinical application, special noise treatment and reconstruction processing methods must be adopted.

In this paper, we investigate a spatially-adaptive temporal smoothing method to alleviate the problems of noise in cardiac gated SPECT sequences. Our previous work presented a Karhune-Loève (KL) transform approach to de-correlate the signals along time axis and obtained satisfied results [1,2]. However, since the heart motion in a full circle varies distinguishably from phase to phase, the KL transform implemented over the full circle will inevitably introduce errors and which may lead to the blur of the images or even lost of detailed information such as minor defects. Considering the nature of cardiac motion, in this study, we classified the gated cardiac SPECT images into three phases: systole, diastole and transition period. Each phase was composed by those sequences share similar time activity. KL transform were then implemented over each group to address the signal relationship among those frames. More than just treating the signal dependant Poisson distributed noise alone in the KL domain, this study also includes treatments toward other degradation factors that pollute the image quality for quantitative SPECT reconstruction. These factors include photon attenuation due to absorption and Compton scatter of primary photons, depth-dependent detector response variation, and scattered photons in the measurement. All these treatments are all incorporated in our proposed four-dimensional analytical frame work. In this frame work, de-correlated components in KL domain were treated sequentially by well studied algorithms to correct and/or compensate for degradation factors mentioned above. Since we have proved that the noise property as well as the transmitted system model remains same in KL domain, those algorithms should be reliable and obtain at least the same performance as they did in spatial domain for each target[3].

This paper is organized as follows: In section II, we introduced our preprocessing method to separate gated sequences into three phases, and proved that the noise model together with the transmitted system model in KL domain remain same as in spatial domain. In addition, we described the algorithms used to improve the image quality in KL domain. Simulation study to evaluate the proposed framework and conclusions are presented in section III and IV separately.

II. METHOD

A. Preprocessing

In one of our previously proposed methods, smoothing of noisy projection data is performed along time axis over the whole obtained sequences based on the KL transform. The covariance matrix was estimated from all the projection sequences collectively by assuming the time-activity curve to be drawn from the same probability density function. Though this approach obtained satisfied results, carefully observation over the reconstruction showed that some detailed information might be blurred or even lost. In some cases, this information plays an important role for clinical treatment. Our previous work presented an improvement by grouping those frames that share similar time-activity curves, which is based on observation, and applying different KL strategy on each group. The reconstruction show a better performance compared with single KL strategy. In this work, we further refine the multi-KL strategy by modeling the cardiac motion and use the adaptive clustering method for grouping. The implementation of automatic classification of the gated frames into three groups could be described as:

a. Initialize and index all the frames as different group;

b. Subtract any two frames pixel by pixel throughout the whole image. Record the indexes of the frames and calculate the total summation of the absolute value as the result.

c. Compare the summation and find the minimum value, and combine the two groups into one according to the recorded indexes.

d. Repeat step b and c until the remained number of group equal to three.

B. KL Transform and System Model

The acquired dynamic projection data sequence y from gated cardiac SPECT can be expressed mathematically as the attenuated Radon transform of the source distribution function λ, which reflects the mean number of gamma photons emitted by a radiotracer injected into the patient's body. The dynamic imaging procedure can be simply modeled as [4-6]:

$$E[y] = H\lambda \tag{1}$$

where H is the SPECT system matrix in spatial domain.

The temporal KL transform of gated sequences could be expressed by:

$$A = M \cdot \lambda \tag{2}$$

Here M is the KL transmission matrix and A is the transmitted independent components in KL domain.

To express the temporal KL transform of the entire dynamic sequence in a matrix format, we define M_L by:

$$M_L = M \otimes I_L \tag{3}$$

where I_L is the $L \times L$ identity matrix and $\otimes$ is the Kronecker product. By multiplying M_L to both sides of the system model (1) and following the same schedule given by [6], we have:

$$\begin{aligned} M_L E[y] &= M_L H\lambda = (M \otimes I_L)(I_K \otimes H_1)\lambda \\ &= (MI_K) \otimes (I_L H_1)\lambda = (I_K M) \otimes (H_1 I_N)\lambda \\ &= (I_K \otimes H_1)(M \otimes I_N)\lambda = HM_N\lambda . \end{aligned} \tag{4}$$

If we define KL transformed data with:

$$\tilde{\lambda} = M_N \lambda, \quad an\,d \quad \tilde{y} = M_L y \tag{5}$$

Then the relationship between transformed gated projection data and transformed image sequence can be reflected by:

$$E[\tilde{y}] = H\tilde{\lambda} \tag{6}$$

Please note that (10) has the same form as (1), which indicates that the KL domain model is the same as that in the original (or spatio-temporal) domain. This means that the system matrix H is exactly the same in both situations, and thus any well-studied reconstruction algorithms could be applied directly in KL domain without modification.

C. Noise Model

It is well recognized that the noise propagation obeys Poisson distribution in SPCET imaging. For the purpose of studying the noise property in KL domain, we simplify the problem by applying the KL transform to those "pure noise" pixels along time sequences, as follows:

By expanding the KL transformation matrix $M_{L,}$ and applying the KL transform to noise signals N, which is also organized as time sequences along time axis, we got:

$$\begin{aligned} M_L \times N &= \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,l} \\ m_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ m_{l,1} & \cdots & \cdots & m_{l,l} \end{bmatrix} \times \begin{bmatrix} n_1 \\ n_2 \\ \cdots \\ n_l \end{bmatrix} \\ &= \sum_{i=1}^{l}\sum_{j=1}^{l} m_{i,j} n_j = \begin{bmatrix} \sum_{j=1}^{l} m_{1,j} n_j \\ \sum_{j=1}^{l} m_{2,j} n_j \\ \cdots \\ \sum_{j=1}^{l} m_{l,j} n_j \end{bmatrix} \end{aligned} \tag{7}$$

It is noted that $m_{i,j}$ in (7) is Poisson distributed and independent with each other. According to the statistic property of Poisson random, the conclusion could be drawn that the KL transmitted element should follow the same distribution with mean value equal to $\sum_{j=1}^{l} m_{l,j} n_j$.

D. Compensation and Reconstruction in KL domain

Based on our previous work, the KL domain 2D smoothing strategy could be expressed firstly by applying the Anscombe transform to all the principle components, which converts Poisson distributed noise into Gaussian distributed one with a constant variance. That is, if x is Poisson distributed with mean equal to λ, then $y = (x + 3/8)^{1/2}$ can be approximated as Gaussian distributed with mean equal to $(\lambda + 1/8)^{1/2}$ and variance of 0.25. Therefore, the noise becomes signal independent and can be expressed mathematically as an additive term. Then a well designed Wiener filter, expressed by (8), can be used to smooth the noise accurately:

$$H(\omega_s,k_\theta)=\frac{S_{pp}(\omega_s,k_\theta)}{S_{xx}(\omega_s,k_\theta)}=\frac{S_{xx}(\omega_s,k_\theta)-S_{nn}(\omega_s,k_\theta)}{S_{xx}(\omega_s,k_\theta)} \quad (8)$$

where $S_{xx}(\omega_s,k_\theta)$, $S_{pp}(\omega_s,k_\theta)$ and $S_{nn}(\omega_s,k_\theta)$ represents the power of acquired data, ideal data and noise data respectively. Noise treated components can then be recovered by inverse Anscombe transform.

After suppression of Poisson noise, the scatter contribution to the primary energy window measurements is then removed by our scatter estimation method, which is based on the detection energy spectrum and modified from the triple-energy-window acquisition protocol. The scattered photon counts within the main window can be described as the following equation:

$$C_s = \frac{2}{3}\times\left(\frac{C_l}{w_l} - x\cdot\frac{C_h}{w_h}\right)\times w \quad (9)$$

where w_l, w_h, w are the width of lower, higher and center (main) energy windows, C_l, C_h are the detector photons from the lower and higher energy window, and C_s is the estimated scatter photons for the main energy window. x is the adjusted parameter.

For correction of detector response, the inversion methods proposed up to date can provide either approximate solutions that realistically characterize the resolution kernel in a real SPECT system, or exact solutions that approximate the resolution kernel to some special functional forms in order to satisfy the mathematical derivations. Our previous study showed that an accurate consideration of the measured resolution kernel is needed to demonstrate robust performance and artifact-free reconstruction and, therefore, is a better choice for quantitative SPECT imaging. In this study, the resolution variation is corrected by the depth-dependent deconvolution, which, based on the central-ray approximation and the distance-frequency relation, deconvolves the scatter-corrected data with the accurate detector-response kernel in frequency domain.

Let $P(l,\omega_t)$ and $\tilde{P}(l,\omega_t)$ be the 2D Fourier transform (FT) of the sinogram $\{p_i\}$ and the deblurred sinogram $\{\tilde{p}_i\}$ respectively, where l is the angular frequency and ω_t is the spatial frequency, the distance-frequency relation is expressed as:

$$P(l,\omega_t) = H(-l/\omega_t,\omega_t)\tilde{P}(l,\omega_t) \quad (10)$$

Where $H(d,\omega_t)$ is the 1D FT of the 2D detector-response kernel h at depth d. The deconvolution is then performed in frequency domain by:

$$\tilde{P}(l,\omega_t) = H^{-1}(-l/\omega_t,\omega_t)P(l,\omega_t) \quad (11)$$

Non-uniform attenuation compensation could be achieved through FBP-type reconstruction based on Novikov's explicit inversion formula with realistic human thoracic attenuation map.

Let (x,y) be the stationary coordinate in the image domain and (t,θ) be the rotation coordinate in the sinogram space. As shown in the last paragraph of Section B, the Novikov's formula could be used directly in the KL-domain. Following the analysis in [7], the KL domain Novikov's inverse formula can be expressed as:

$$\phi(\vec{r}) = \frac{1}{4\pi} div \int_0^{2\pi} \vec{j}[\exp([D\mu]_\theta(s,t))\tilde{q}(t,\theta)]\Big|_{\substack{s=\vec{r}\bullet\vec{j}\\ t=\vec{r}\bullet\vec{k}}} d\theta \quad (12)$$

where $\vec{j} = (\cos\theta, \sin\theta)$, $\vec{k} = (-\sin\theta, \cos\theta)$, ***div*** is the divergence operation, $\phi(\vec{r})$ is the reconstructed image frame from its corresponding sinogram data frame $A(t,\theta)$ in the KL domain and

$$\tilde{q}(t,\theta) = e^{-h_1}\{\cos(h_2)\tilde{q}_1(t,\theta) + \sin(h_2)\tilde{q}_2(t,\theta)\} \quad (13)$$

$$\tilde{q}_1(t,\theta) = \hat{H}\cos(h_2)e^{h_1}A(t,\theta) \quad (14)$$

$$\tilde{q}_2(t,\theta) = \hat{H}\sin(h_2)e^{h_1}A(t,\theta) \quad (15)$$

with $h_1 = \frac{1}{2}[R\mu](t,\theta)$, $h_2 = [\hat{H}h_1](t,\theta)$. The operators $\hat{H}$, D, and R represent the Hilbert transform, the divergent beam transform, and the Radon transform, respectively, and are defined as follows:

$$[\hat{H}g](s) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{g(\tau)}{s-\tau}d\tau \quad (16)$$

$$[D\mu]_\theta(s,t) = \int_t^{\infty}\mu_\theta(s,\tau)d\tau \quad (17)$$

$$[R\mu](s,\theta) = \int_{-\infty}^{\infty}\mu_\theta(s,t)dt \quad (18)$$

The above quantitative reconstruction is performed frame-by-frame in the KL domain for each principal component, which is similar to that performed in the spatial domain, and the result is $\Phi_{m,n} = (\phi^1_{m,n}, \phi^2_{m,n}, \cdots \phi^l_{m,n}, \phi^{l+1}_{m,n}, \cdots, \phi^K_{m,n})$ for each image pixel (m,n) in the KL domain. Since the higher-order components with smaller eigenvalues may have little information, only the resulted frames reconstructed from the first l low-order components, i.e. $\Phi^l_{m,n} = (\phi^1_{m,n}, \phi^2_{m,n}, \cdots \phi^l_{m,n})$ ($l \le K$), could be retained for further noise reduction and computing efficiency. After Novikov's inversion in the KL domain, an inverse KL transform on the K or l reconstructed frames will generate the gated images in the original space:

$$\hat{\lambda}_{m,n}^{time} = M^{T} \Phi \quad (19)$$

III. RESULTS

The presented methods above were tested by computer simulations using the chest digital phantoms of 128cubic array and the clinical data of 64 cubic array, which mimic the radiotracer distribution of myocardial perfusion in gated frames. Parallel-beam projections of $128^2/64^2$ size were evenly spaced over $360^o/180^o$ for 8/8 frames.

Final results could be achieved by inverse KL transform the processed components. Multi-degradation factors were considered in this study. Both the simulation study on digital phantom and clinical data achieved encouraging results. The degradation factors added were compensated or corrected and achieved the similar or even better performance compared with any single treatment in spatio-temporal domain.

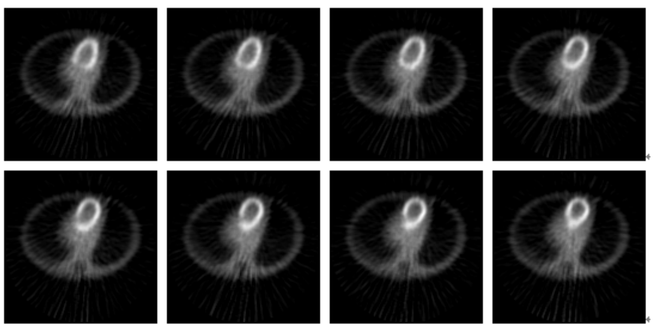

Figure 1: Test results of digital phantom from the presented adaptive analytical approach.

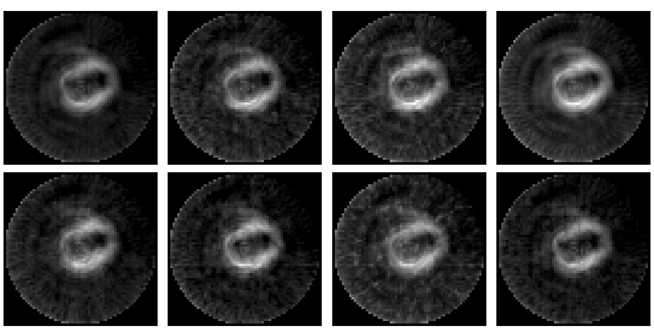

Figure 2: Test results of clinical data from the presented adaptive analytical approach.

IV. CONCLUSION

In this paper, we further improve our previous analytical reconstruction scheme for quantitative gated SPECT. To fully utilize the de-correlation advantages of KL transform, the gated sequences were fist classified into three groups and then each of them was treated by different KL strategy. In the KL domain, well studied algorithms were used step by step, aiming to improve the image quality by the correction of the degradation factors carefully. Quantitative inversion could be achieved through corresponding inverse KL transform.

The presented compensation and reconstruction scheme seeking for an exactly analytical approach thus could achieve quantitative results theoretically. Other than some research work focusing on noise reduction alone by addressing the signal correlation property, this study fully utilized the well studied algorithms in SPECT imaging and offered a unique way to treat all major degradation factors in one framework. The simulation study validates its potential for gated SPECT imaging.

However, due to the inevitable errors introduced by computational implementation of algorithms, and, especially when these errors might be spread or even enlarged by the sequential compensation strategy, the final result may not as stable as some established iterative approaches, such as OS-EM. Further works is needed to improve the performance and robustness of the proposed framework.

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