Ray-driven Analytical Fan-beam SPECT Reconstruction with Non-uniform Attenuation

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Abstract: Single-photon emission computed tomography (SPECT) is based on the measurement of radiation emitted by a radiotracer injected into the patient. Because of photoelectric absorption and Compton scatter, the γ photons are attenuated inside the body before arriving at the detector. A quantitative reconstruction must consider the attenuation, which is usually non-uniform. Novikov and Natterer had derived an explicit formula for non-uniformly attenuated SPECT reconstruction of parallel-hole collimators. In this paper, we extend their researches to fan-beam collimators. Because any ray in fan-beam geometry can be treated as a ray in parallel-beam geometry, we derived a ray-driven analytical fan-beam reconstruction formula with non-uniform attenuation. Its accuracy is demonstrated computer simulation experiments*.

1 Introduction

Single-photon emission computed tomography (SPECT) reconstructs the image of a radiotracer or radiopharmaceutical uptake distribution within the body from the measurement of decayed γ ray radiation, where the radiotracer is injected intravenously into the patient and the uptake image at a region of interest (ROI) reflects directly the cell function there. Because of photoelectric absorption and Compton scatter, the γ photons are attenuated inside the body before arriving at the detector. Tretiak and Metz [8] developed an explicit inversion formula of uniform attenuated (or exponential) Radon transform in two dimensions for parallel-hole collimator geometry. This algorithm assumes that the attenuation coefficient is constant across the body and that the body contour is convex. Some alternative inversion algorithms were developed late with extension to more complicated collimator geometries. Weng *et al.* [6] extended the parallel-beam algorithm to fan-beam collimator geometry by a coordinate transform. You *et al.* [4] derived a Cormack-type inversion of the exponential Radon transform by employing the circular harmonic transform directly in both the projection space and the image space, instead of the Fourier space. It was applied to parallel-hole, fan-beam and variable focal-length fan-beam collimator geometries. However, all of these algorithms are limited to uniform attenuation, which is not applicable for imaging of non-uniform attenuation, such as cardiac studies, where a quantitative reconstruction must consider the non-uniform attenuation among the lungs, soft tissues and rib bones.

Novikov and Natterer [1, 3] had derived an explicit inverse formula for non-uniform attenuation Radon transform of parallel-hole collimators. Their formula had been implemented and good reconstruction results were obtained. For clinical applications, however, fan-beam and variable focal length fan-beam collimators would be preferred for brain and chest imaging. Fan-beam collimator is used to improve the sensitivity and resolution for imaging small organs like the brain. Variable focal-length fan-beam collimator overcomes the truncation problem of fan-beam collimator when

* This work was supported in part by NIH National Heart, Lung and Blood Institute under Grant No. HL54166.

a small organ is embedded in a large body, such as the heart in the chest. In this work, we extend their previous researches to fan-beam collimators. The underlying principle can be applied to variable focal-length fan-beam geometry. Because any ray in fan-beam geometry can be treated as a ray in parallel-beam geometry, we derived a ray-driven analytical fan-beam reconstruction formula with non-uniform attenuation, based on the parallel-hole non-uniform attenuation reconstruction formula and the relation between the parallel-hole geometry and fan beam geometry.

2 Basic Notation

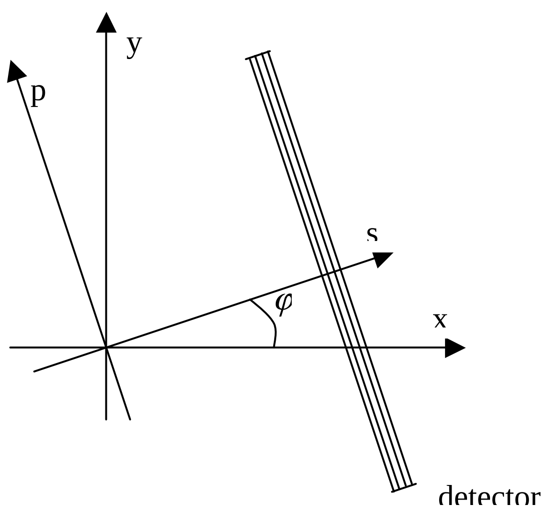

Fig. 1: The rotated coordinate system

The notation of [2] will be used throughout this paper. Let $f(x)$ denote the object function to be reconstructed and $a(x)$ be the attenuation coefficient across the body. Let $g_\varphi(p)$ be the projection datum at position p with projection angle $\theta(\varphi)$. In order to simplify the derivation, a rotated coordinate system (s, p) is introduced,

$$\begin{aligned} s &= x\cos\varphi + y\sin\varphi \\ p &= -x\sin\varphi + y\sin\varphi \end{aligned} \qquad (1)$$

then

$$g_\varphi(p) = \int_{-\infty}^{\infty} \exp(-D_\varphi a_\varphi(s,p)) f_\varphi(s,p)ds \qquad (2)$$

where the divergent beam transform

$$D_\varphi a_\varphi(s,p) = \int_s^{\infty} a_\varphi(s,p)ds \qquad (3)$$

3 Reconstruction for Parallel-hole Collimator

According to [1, 2, 3], parallel-hole reconstruction formula can be written as follows,

$$f(x) = \frac{1}{4\pi}\int_0^{2\pi} \frac{\partial}{\partial p}(e^{(Da)(s,p)} ga(p))d\varphi \qquad (4)$$

where

$$ga(p) = e^{-A(p)}[\cos(HA(p))H\left(\cos(HA(p))e^{A(p)}g(p)\right) + \sin(HA(p))H\left(\sin(HA(p))e^{A(p)}g(p)\right)]$$

$$A(p) = \frac{1}{2}Ra(p), \qquad (5)$$

and H denotes the Hilbert transform [2] and R represents the Radon transform [2].

So, parallel-beam reconstruction process can be described as follows

a) Computing the divergent beam transform $D_\varphi a_\varphi(s,p)$ and the Radon transform $Ra(p)$.
b) Computing the modified projections $ga(p)$.
c) Differentiating the result of $e^{(Da)(s,p)} ga(p)$ in p.
d) Performing back-projection.

4 Reconstruction for Fan-beam Collimator

Any ray (p,β) in fan-beam geometry can be seen as a ray (x_r,θ) in parallel-hole geometry. D is the focus length. The relation between two geometries is:

$$\theta = \beta + \gamma = \beta + tg^{-1}\frac{p}{D}$$

$$x_r = p\cos\gamma = \frac{pD}{\sqrt{D^2 + p^2}} \qquad (6)$$

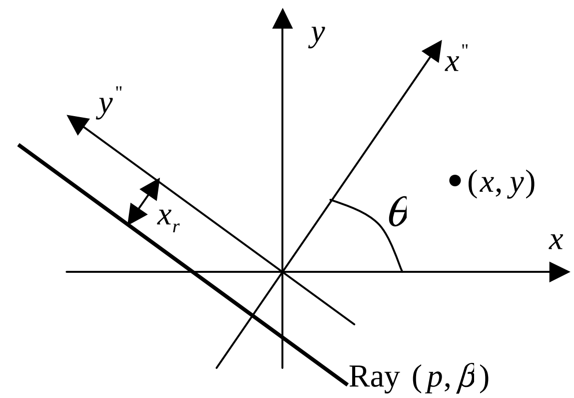

Fig. 2: The local coordinate system.

For each ray (p,β), we can build a local coordinate system (x'', y''). The relation between this local coordinate system and original coordinate systems is:

$$\begin{aligned} x'' &= x\cos\theta + y\sin\theta \\ y'' &= -x\sin\theta + y\cos\theta \end{aligned}. \qquad (7)$$

In this local coordinate system (see Fig. 2), this ray will be a parallel-beam ray. So we can use the parallel-beam formula to process this ray. For any point (x, y), its position in this local coordinate system is (x'', y''). A ray-driven analytical fan-beam reconstruction formula can be written as follows

$$f(x,y) = \frac{1}{4\pi}\int_0^{2\pi} M_\theta(x,y)d\theta \qquad (8)$$

where

$$M_\theta(x,y) = \frac{\partial}{\partial x''}(e^{(Da)(x'',y'')} \cdot ga(x''))$$

$$ga(x'') =$$
$$e^{-A(x'')}[\cos(HA(x''))H(\cos(HA(x_r))e^{A(x_r)}g(p,\beta)\delta(x''-x_r))$$
$$+\sin(HA(x''))H(\sin(HA(x_r))e^{A(x_r)}g(p,\beta)\delta(x''-x_r))]$$

$$A(x'') = \frac{1}{2}Ra(x'') \qquad (9)$$

and H and R have been defined before. The relation between (x, y) and (x'', y'') is showed by equation (7). The relation between (p,β) and (x_r,θ) is showed by equation (6).

5 Simulations

An experimental study was carried out to test the derived formulas using the Shepp-Logan mathematical phantom with non-uniform attenuation on an image array of 128x128 size. From the reconstruction result without attenuation compensation, we can see the strong effect of non-uniform attenuation. Setting the focus length as ∞, which means a parallel-beam geometry, the reconstruction result is the same as that reconstructed by non-uniform attenuation parallel-beam reconstruction formula. With fan-beam collimators of focal lengths of 300 pixel units, we also obtained identical result as the original image. Otherwise, by setting the attenuation coefficients to be zero, the images that reconstructed by our algorithm were identical to those of the conventional filtered back-projection. It proves the correctness of our computer coding, (see Fig. 3).

6 Discussions and Conclusion

From Fig. 3, we can see that there are some artifacts in the reconstruction image, and the artifacts in fan-beam reconstruction image are more than those in the parallel-beam reconstruction image. These artifacts are the cause of the error of interpolation. There are more non-parallel rays in the fan-beam collimator geometry than those in parallel-beam collimator geometry, which lead to more interpolation error and more artifacts in fan-beam collimator geometry.

Our method is ray driven. It computes the results by a ray-by-ray manner. This may cost more computing time, as compared to the parallel-hole reconstruction algorithm. But this method is an exact analytical fan-beam reconstruction formula. The experiments demonstrated the accuracy of the derived formulas for analytical reconstruction of non-uniformly attenuated SPECT with fan-beam collimators.

References:

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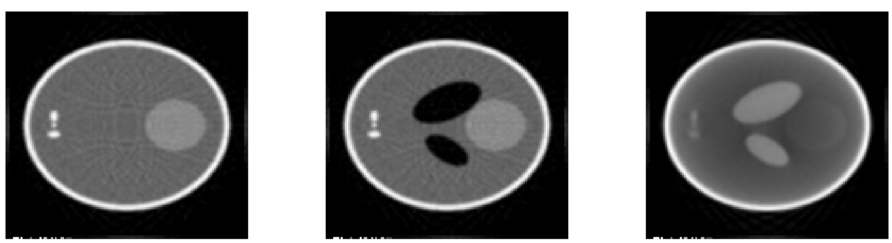
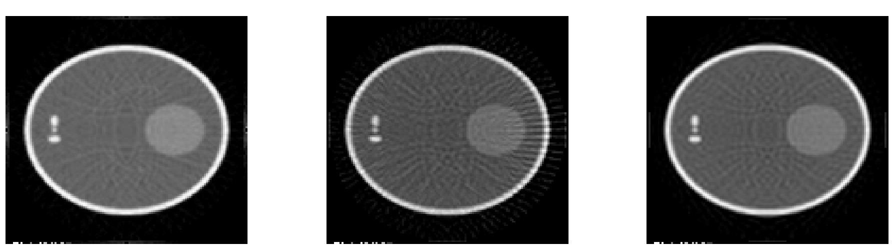


Activity Phantom. Attenuation Map. Reconstruction without attenuation compensation.



Parallel-beam FD = ∞. Fan beam FD = 300. Fan beam FD = 300 from non attenuated projections.

Fig. 3: Reconstruction results using our method
(FD is focus length, unit is pixel, image size is 128×128)