A Unified Reconstruction Algorithm for Conventional 2-D Acquisition Geometry*

Jiangsheng You, Zhengrong Liang and Jianxin Cai†
Department of Radiology, State University of New York, Stony Brook, NY 11794
†Institute of Heavy Ion Physics, Peking University, Beijing 100871, P. R. China

Abstract

To improve detection sensitivity, fan- and general converging-beam collimations are usually used in SPECT imaging. Development of the associated reconstruction algorithms is currently a research topic. In this paper, we present a unified harmonic decomposition reconstruction algorithm for general converging-beam geometry, including parallel- or fan-beam collimation as a simplfied example. Compared to other harmonic transform algorithms, the presented one has the advantage that the tedious computation of the Hankel transform, the Chebyshev polynomials or the Bessel functions is not necessary. In fact, the current algorithm can be regarded as an alternative implementation of the conventional FBP algorithm in terms of harmonic decomposition. The implementation is easy and the computation time is comparable to the FBP algorithm. The reconstructed images have the same quality as that of the FBP. In addition to the advantages above, the developed algorithm is able to invert the uniformly attenuated Radon transform which has a significant impact in quantitative SPECT imaging.

§1 Introduction

The basics of filtered backprojection (FBP) algorithm was described by Shepp and Logan [1]. Herman and Naparstek extended the FBP algorithm for fan-beam geometry [2]. Tretiak and Metz obtained a FBP algorithm for projections of parallel-beam geometry with uniform attenuation by the inversion of the exponential Radon transform [3]. Hawkins et al used the circular harmonic transform to reconstruct the image based on some relations in frequency domain [4]. We have studied the reconstruction problem for complicated converging-beam geometry [5,6], using an alternative implementation of FBP by the harmonic decomposition in the projection space rather than in the frequency domain of [4]. In this paper, we present a general procedure to study the conventional acquisition geometry with and without considering the uniform attenuation.

*This work was supported by NNSF of China under Grant 39570223 and Grant 19675005, in part by NIH Grant HL 51466 and Grant NS 35853, and EI Award of American Heart Association.

§2 Acquisition Geometry

The parallel- and converging-beam geometries are shown in Fig.1 and Fig.2, respectively. When the focal length of the converging-beam is constant, it becomes the conventional fan-beam geometry. When the focal length becomes infinite, a parallel-beam collimation is obtained.

§3 FBP and Its Harmonic Decomposition Implementation

Without loss of generality, we only consider the projections with uniform attenuation. If the attenuation coefficient μ_0 becomes zero, the (inverse) exponential Radon transform becomes the standard (inverse) Radon transform. In the parallel-beam geometry, the FBP is expressed by the following form:

$$f(r,\ \varphi) \approx \frac{1}{2}\int_0^{2\pi} d\theta e^{2\pi\mu_0 r\sin(\theta-\varphi)} \int_{-\infty}^{\infty} W(r\cos(\theta-\varphi)-s)P_\mu(s,\ \theta)ds \tag{1}$$

where $P_\mu(s, \theta)$ denotes the exponential Radon transform [3], μ_0 denotes the constant attenuation coefficient, and $W(\cdot)$ denotes a convolution kernel function. For detailed discription, see [1,3]. The

0-7803-4261-5/98/$10.00 (c) 1998 IEEE

harmonic decomposition implementation of Eq.(1) can be expressed as follows:

$$f(r,\varphi) = \sum_{-\infty}^{\infty} f_n(r) e^{in\varphi} \quad (2)$$

$$f_n(r) = \frac{1}{2} \int_{-\infty}^{\infty} ds \int_{0}^{2\pi} d\theta e^{-in\theta} P_\mu(s,\theta) \int_{0}^{2\pi} e^{in\varphi} W(r\cos\varphi - s) e^{2\pi\mu_0 r \sin\varphi} d\varphi. \quad (3)$$

Under the expressions of (2) and (3), the integral of (1) can be implemented separately in the polar coordinate. Because of the rotating nature of the acquisition geometry, (2) and (3) can be easily used to study the reconstruction problem for general converging-beam geometry. The coordinate transforms between parallel-beam and converging-beam geometry are expressed, see Fig.1 and Fig.2, as

$$s = D(\alpha)\sin\alpha \qquad \theta = \Phi + \frac{\pi}{2} + \alpha. \quad (4)$$

Then the exponential Radon transform in the converging-beam geometry satisfies the relation

$$P_\mu(\alpha,\, \Phi) = P_\mu(s,\, \theta). \quad (5)$$

Because α is one-to-one corresponding to s, so for fixed α, $P_\mu(\alpha,\, \Phi)$ and $P_\mu(s,\, \theta)$ stand for the same periodic function. Then Eq.(3) can be easily implemented. The detailed procedure can be summarized as follows,

1. for every fixed α, compute the FFT (fast Fourier transform) of $P_\mu(\Phi,\, \alpha)$ with respect to the variable Φ;

2. let $\tilde{P}_\mu(\omega,\, \alpha)$ be the FFT of $P_\mu(\Phi,\, \alpha)$, then translate $\tilde{P}_\mu(\omega,\, \alpha)$ by the factor $e^{-i(\frac{\pi}{2}+\alpha)\omega}$ (this step is not needed for parallel-beam geometry);

3. compute the inverse FFT of filter function $W(r\cos\varphi - s)e^{2\pi\mu_0 r\sin\varphi}$ and multiply it by the result of step 2;

4. integrate with respect to the variable α;

5. compute the inverse FFT of the results of step 4 to obtain the reconstructed image $f(r,\varphi)$ in the polar coordinate system;

6. transform the reconstructed image from polar coordinate $f(r,\varphi)$ to Cartesian coordinate $f(x,y)$.

§4 Numerical simulations

The reconstructed images by the presented algorithm are shown in Figs.3-8. In these simulations, the Shepp-Logan phantom was used to generate the projection data. Then the FBP algorithm and the presented algorithm were used to reconstruct the images. All discrete grids have the same size of 128×128.

§5 Conclusion

The presented harmonic decomposition algorithm possesses a unified form for the conventional acquisition geometry and can avoid the complicated computation of the special functions in the circular harmonic transforms of [3,4]. Because we can use the FFT for the angle variable, so the computation speed can be comparable to the FBP algorithm. The detailed comparison results are given in [5,6]. The current algorithm greatly reduces the complexity of reconstruction for the general converging-beam acquisition geometry.

References

[1] L. A. Shepp and B. F. Logan, "The Fourier reconstruction of a head section," *IEEE Trans. Nucl. Sci.*, vol. 21, pp. 21-43, 1974.

[2] G. T. Herman and A. Naparstek, "Fast image reconstruction based on a Radon inversion formula appropriate for rapidly collected data," *SIAM J. Appl. Math.*, vol 33, pp. 511-533, 1977.

[3] O. Tretiak and C. Metz, "The exponential Radon transform," *SIAM J. Appl. Math.*, vol. 39, pp. 341-354, 1980.

[4] W. G. Hawkins, P. K. Leichner and C. N. Yang, "The circular harmonic transform for SPECT reconstruction and boundary conditions on the Fourier transform of the sinogram," *IEEE Trans. on Med. Imag.*, vol 7, pp. 135-148, 1988.

[5] J. You, Z. Liang and S. Bao, "A harmonic decomposition reconstruction algorithm for spatially varying focal length fan-beam collimators," submitted for publication.

[6] J. You, Z. Liang and G. Zeng, "A unified harmonic decomposition inversion of the exponential Radon transform for parallel- and variable focal length fan-beam acquisition geometries," submitted for publication.

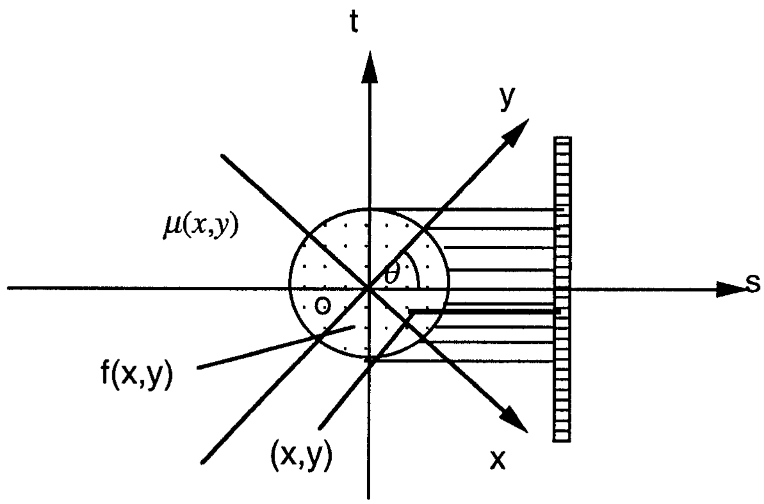

Fig.1. Parallel-beam geometry

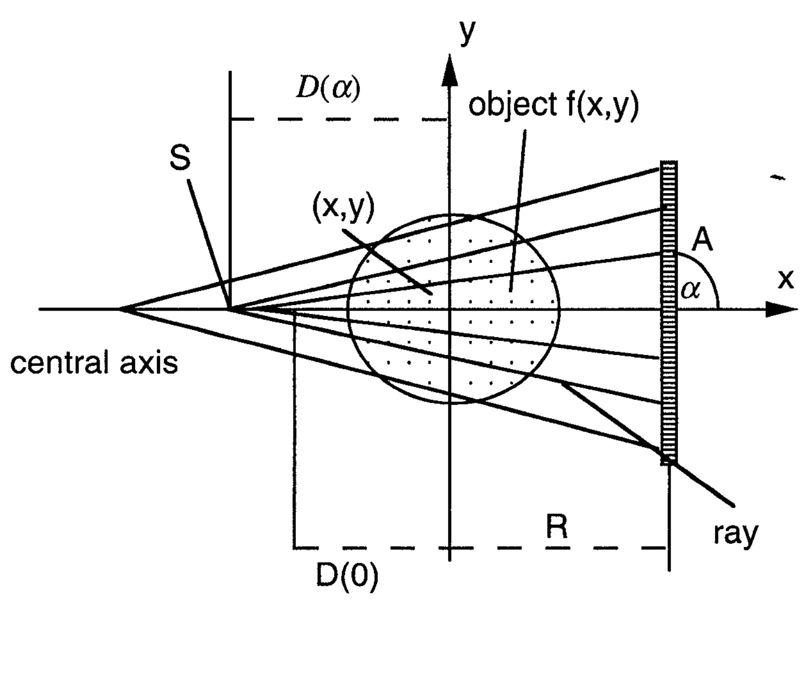

Fig. 2. Converging-beam geometry

Fig. 3. The Shepp-Logan phantom

Fig. 4. The reconstructed image by FBP for fan-beam data without attenuation.

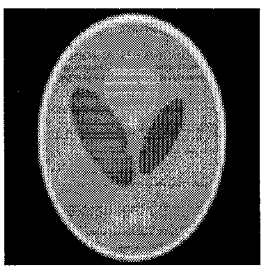

Fig. 5. The reconstructed image by the presented algorithm for fan-beam data without attenuation.

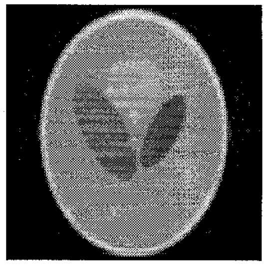

Fig. 6. The reconstructed image by the presented algorithm for converging-beam $D(\alpha) = 3.0/\cos\alpha$, without attenuation.

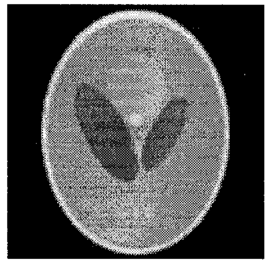

Fig. 7. The reconstructed image by the presented algorithm for fan-beam data with $\mu_0 = 0.15$ /cm.

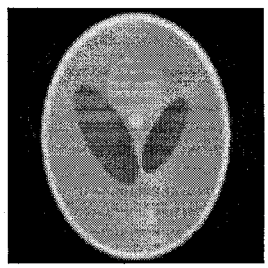

Fig. 8. The reconstructed image by the presented algorithm for converging-beam $D(\alpha) = 3.0/\cos\alpha$, with $\mu_0 = 0.15$ /cm.