The Possibility of Complete Restoration of Variable Collimator Response in SPECT Imaging

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Abstract— **Spatial resolution of SPECT imaging is mainly limited by the geometry of collimator holes. A long collimator hole with a small diameter improves the spatial resolution at the cost of low detection efficiency. With a finite size for the holes, the resolution changes as a function of the distance from the collimator surface. Lewitt *et al* developed an approximate restoration algorithm to correct for the distance-dependent collimator blurring by the Fourier transform of sinogram. Van Elmbt *et al* presented an analytical formula assuming an approximated resolution variation function. In this work, an accurate restoration procedure is investigated for a fan integral projection data based on the collimator hole geometry.**

The variable collimator response is illustrated as a fan-region integral of the radionuclide distribution, which is different from the conventional line integral, when using the FBP algorithm to reconstruct the image. In a parallel-beam acquisition geometry, if the radionuclide distribution is restricted to a bounded region, the line integral can be accurately computed from the fan integral near the boundary, then by a bootstrap technique, all these line integrals can be obtained. Therefore the variable collimator response can be completely restored theoretically.

Index Terms-**Fan integral, line integral, collimator blurring.**

I. Introduction

Collimator is used in single photon emission computed tomography (SPECT) to allow photons from a limited range of directions to pass through to the scintillation crystal and thus provides approximately the emission locations of the detected photons along the angular range. Reconstruction is performed to determine the emission intensity (or radionuclide) distribution in the human body. The reconstructed image has usually a low resolution with blurred details. As pointed out in Chapter 16 of [1], the resolution blurring is mainly due to two factors: collimator characteristics, detector and positioning electronics. At a distance of 5∼10 cm (a typical depth of organs inside the body), the resolution blurring is dominated primarily by the collimator characteristics.

Conventional approaches assume that the collimator response is spatially-invariant. Then standard inverse filtering algorithms and restoration techniques can be used to correct the blurring [2]. Based on a specific model, Ogawa *et al* presented an analytical method utilizing the Fourier transform (FT) of sinogram [3]. Lewitt *et al* developed an approximated distance-dependent inverse filter [4,5,6]. Recently, investigations were made based on various approximations to the blurring kernel [7,8,9]. If the projection data are assumed to be fan integrals, instead of line integrals within the resolution kernel, for the collimator blurring, an accurate restoration is feasible. The restoration determines the line integral from the fan integral. The presented theory below indicates that reconstruction from fan integrals can be accurately performed in the same manner as reconstruction from line integrals.

II. The Collimator Response

The variable collimator and electronic response in detection is one major factor limiting quantitative SPECT imaging, in addition to the photon attenuation (absorption and scatter) in the body. Here we will concentrate on the problem of collimator response restoration.

Assume that the radionuclide distribution is denoted by a two-dimensional (2D) function $f(x, y)$. Usually $f(x, y)$ is assumed to be a compactly smooth continuous function because the radionuclide distribution is always located in a bounded region. Let the support region of $f(x, y)$ be inside a disc $B(r)$ with center at the origin and radius r.

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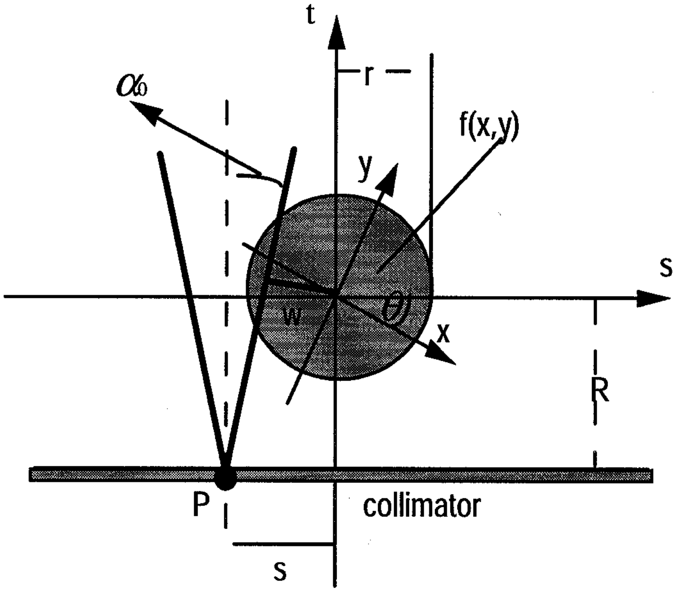

Fig. 1. The radionuclide distribution f(x,y) is located in the disc B(r), the collimator is away from the origin at a distance R. The total photons through the collimator hole P is an integral inside the fan-like region.

Let coordinate (s, t) be the rotated frame with respect to the stationary coordinate (x, y), as shown in Fig.1. Obviously,

$$\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \quad (1)$$

The Radon transform $p(s, \theta)$ of $f(x, y)$ is expressed as

$$p(s,\theta) = \int f(s\cos\theta - t\sin\theta, s\sin\theta + t\cos\theta)dt. \quad (2)$$

As shown in Fig.1, an acquired projection datum at P is not exactly a line integral as defined by (2). For a specific collimator, see Fig.2, the arriving photons on the crystal can be regarded as the total emissions from the fan-like region. According to [6], the acquired datum by the collimator hole centered on the ray (s, θ) is a fan integral $p_R(s, \theta)$ of radionuclide distribution, and can be expressed as

$$p_R(s,\theta) = \int [\int f(s',t';\theta)h(s-s',t'+R)ds]dt' \quad (3)$$

where $f(s', t'; \theta) = f(s'\cos\theta - t'\sin\theta, s'\sin\theta + t'\cos\theta)$, $h(\cdot, \cdot)$ stands for the collimator response.

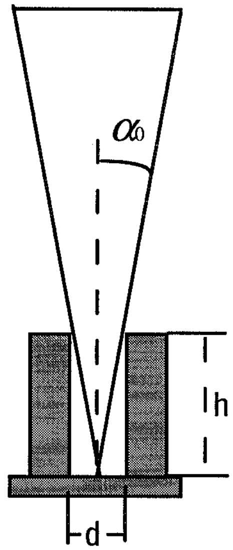

Fig. 2. For a specific collimator, the arriving photons on the crystal can be represented as all the emmisions from the fan region.

Approximation was made in various approaches to estimate $p(s, \theta)$ from $p_R(s, \theta)$ by the FT of the sinogram [6,8].

In this work, we will prove the possibility of an accurate restoration procedure to compute $p(s, \theta)$ from $p_R(s, \theta)$.

III. Derivation of the Line Integral

Without considering the penetration and scattering of the collimator materials, the fan integral $p_R(s, \theta)$ can be rewritten as

$$p_R(s,\theta) = \int dt' \int_{s-(t'+R)\tan\alpha_0}^{s+(t'+R)\tan\alpha_0} f(s',t';\theta)ds'. \quad (4)$$

It is noted that (2) is a 1D integral and (4) is a 2D integral. To establish a relation between (2) and (4), a derivative is needed. By a partial derivative of $p_R(s, \theta)$ with respect to the variable s, we have

$$\begin{aligned} &\frac{\partial}{\partial s} p_R(s,\theta) \\ &= \int (f(s + (t'+R)\tan\alpha_0, t';\theta) \\ &\quad - f(s - (t'+R)\tan\alpha_0, t';\theta))dt'. \quad (5) \end{aligned}$$

Before the left edge of the fan reaches the disk $B(r)$ in Fig.1, when $s < \frac{R\sin\alpha_0 - r}{\cos\alpha_0}$,

$$\int f(s - (t' + R) \tan \alpha_0, t'; \theta) dt' = 0 \quad (6)$$

and

$$\begin{aligned} & \frac{\partial}{\partial s} p_R(s, \theta) \\ = & \int f(s + (t' + R) \tan \alpha_0, t'; \theta) dt' \quad (7) \\ = & \cos \alpha_0 \int f(w, t; \theta - \alpha_0) dt \end{aligned}$$

where $w = s \cos \alpha_0 + R \sin \alpha_0$. This indicates that $p(w, \theta - \alpha_0) = \frac{1}{\cos \alpha_0} \frac{\partial}{\partial s} p_R(s, \theta)$ when $w \in [-r, \quad \delta - r]$, $\delta = 2R \sin \alpha_0$.

Because the reconstruction from line projection is an exterior problem, for details see [10], so we can locally reconstruct the image in the region $\sqrt{x^2 + y^2} > r - \delta$. With a step-by-step procedure, we can reconstruct the whole image $f(x, y)$ in finite steps. As shown by [10], the exterior problem is generally an ill-posed problem and unstable in numerical calculation. In the following, we will present a bootstrap technique to determine all the line integrals $p(s, \theta)$. Then the well established FBP (filtered backprojection) method can be used to accomplish the reconstruction task.

Without loss of generality, we assume that $r = 2K\delta$, where K is an integer. In the above procedure, combining the acquisitions for the angles θ and $\theta + \pi$, we can obtain the line integrals $p(s, \theta - \alpha_0)$ in the intervals $[2k\delta - r, \quad (2k+1)\delta - r]$, and the line integrals $p(s, \theta + \alpha_0)$ in the intervals $[(2k+1)\delta - r, \quad 2(k+1)\delta - r]$ where $k = 0, 2K-1$. When θ rotates from 0 to π, all the line integrals $p(s, \theta)$ are obtained. Then the conventional FBP-based algorithms can be used to reconstruct the image from line integrals.

IV. Discussion

By modeling the collimator response as a fan integral, an accurate restoration procedure can be derived. To obtain the line integrals from the fan integrals for a FBP-based reconstruction, a derivative is required. When using the FBP algorithm to reconstruct the image, another derivative of the line integral is also required. Therefore, a total of two derivatives are needed in an accurate reconstruction. In general, the numerical stability is not warranted. By the bootstrap technique, our theory indicates that an accurate reconstruction is possible.

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