A Theoretically Based Pre-Reconstructing Filter for Spatio-Temporal Noise Reduction in Gated Cardiac SPECT [1]

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Abstract

In dynamic SPECT studies, the acquired sinograms have both spatial and temporal correlations among the time sequence, in addition to the spatial correlation within each time frame (i.e., a three-dimensional (3D) sinogram). In this paper, we propose a theoretically based multi-frame filtering algorithm, which considers both the spatial and temporal correlations, for restoring the gated sinograms degraded by the Poisson noise. This spatio-temporal filtering task is greatly simplified by first applying the Anscombe transformation to all the sinogram data, which converts Poisson distributed noise into Gaussian distributed one with constant variance (i.e., the noise is signal independence). By performing temporal Karhune-Loève (K-L) transformation on the Anscombe transformed data sequence, the filtering task is further simplified from a 4D problem to a 3D spatial process. In the K-L domain, the noise property of constant variance remains for all components, while the signal-to-noise ratio decreases monotonically for lower eigenvalue components. An accurate Wiener filter is then constructed for each component of a 3D data set. By this approach, the spatio-temporal filtering can be achieved at a reasonable computational cost. The computer simulations are very encouraging, by visual judgement, as compared to frame-by-frame 3D Wiener filtering along the time sequence.

I. INTRODUCTION

Dynamic studies on the body functions can provide more rich information than static studies, leading to improved diagnosis in clinic. In gated cardiac SPECT (single-photon emission computed tomography) studies, the acquired sinograms have both spatial and temporal correlations among the time sequence, in addition to the spatial correlation within each time frame, i.e., a three-dimensional (3D) sinogram from a 3D source distribution. Unfortunately, conventional approaches process the data sequence frame-by-frame [1, 2], ignoring the temporal correlation among the time sequence.

Recently, a great research effort has been devoted to process this spatio-temporal data sequence, in order to improve the potential of gated cardiac SPECT for diagnosis of CAD (coronary artery disease), which causes more than 40% deaths (the leading cause of death) each year in this nation. One approach models the time dimension, in addition to the 3D spatial correlation, in a Gibbs prior for a 4D maximum *a posteriori* probability (MAP) reconstruction of the image series [3]. Although it is theoretically attractive in the Bayesian framework, this approach requires valid prior model and intensive computing power. Another approach employs the Karhune-Loève (K-L) transformation along the time dimension and eliminates the lower eigenvalue components in the K-L domain, assuming that these components contain very little useful information [4-6]. This later approach is very effective for noise reduction and, therefore, should be practically useful for the reconstruction of the image series if the assumption that the eliminated components contain little information is valid. In clinic situations, this assumption is often invalid [5, 7].

Besides noisy appearance, it is well known that SPECT images suffer from Poisson distributed noise that varies over different part of the image depending on the intensity at each pixel [8]. This means that the variance of the Poisson noise depends on the signal itself. For accurate treatment of this kind of noise, we need to estimate the statistic properties of noise from signal, which usually is difficult, sometimes may be impossible [8, 9].

In this paper, we propose a theoretically based multi-frame filtering algorithm, which considers both the spatial and temporal correlations, for restoring the gated sinograms degraded by the Poisson noise. It constructs an adaptive 3D Wiener filter for each component in the K-L domain for accurate treatment of the Poisson noise in the 4D dynamic process, leading to an improved reconstruction of the image series.

II. METHODS

A. Background and Problem Formulation

1) Noise Model It is well known that the observed SPECT data $\boldsymbol{g}$ is Poisson distributed with mean equals to the projection of true image intensity [10]. With this model, the SPECT noise suppression problem can be viewed as: for a given single realization of a jointly independent Poisson distributed $\boldsymbol{g}$, how can we estimate its mean $\boldsymbol{p}$. For MMSE (minimum mean square error) filtering, an equivalent and more useful model is to express the noise as an additive signal-dependent term

$$\boldsymbol{g} = \boldsymbol{p} + \boldsymbol{n} \quad (1)$$

where the mean of the noise $\boldsymbol{n}$ equals to $\boldsymbol{0}$ and the variance equals to $E(\boldsymbol{p})$ [8, 10]. Unlike white noise that is signal independent, the variance of Poisson noise depends on the signal itself. This property makes the implementation of Wiener filter more difficult in SPECT because any error resulting from the estimation of noise variance, which is unknown and need to be estimated from noisy data, will

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degrade the accuracy of the filter. If the variance of noise can be determined exactly by an alternative way, the performance of the filter will be improved.

2) Anscombe Transformation The task of spatio-temporal filtering of signal-dependent Poisson noise can be greatly simplified by first applying the Anscombe transformation to all the sinogram data, which converts Poisson distributed noise into Gaussian distributed one with constant variance (i.e., the noise becomes signal independence). That is, if x is Poisson distributed with mean equal to l, then $y = (x+3/8)^{1/2}$ can be approximated as Gaussian distributed with its mean equal to $(l+1/8)^{1/2}$ and its variance equal to 0.25 [11]. Thus, by performing Anscombe transformation, the Poisson noise model can be rewritten as

$$\boldsymbol{g}' = \boldsymbol{p}' + \boldsymbol{n}' \tag{2}$$

where $\boldsymbol{g}' = [g'_1 \ \ g'_2 \ \ldots \ g'_K]^T$, $\boldsymbol{p}' = [p'_1 \ \ p'_2 \ \ldots \ p'_K]^T$, and $\boldsymbol{n}' = [n'_1 \ \ n'_2 \ \ldots \ n'_K]^T$. Notations g'_i, p'_i, and n'_i ($i=1,2,\ldots,K$) denote the $M\times 1$ vectors obtained by lexicographically ordering the transformed observed data, the original intensity, and the noise, respectively, for frame i, while $n'_{i;j}$ is Guassian distributed white noise with mean equals to 0 and constant variance equals to 0.25 at pixel j. Therefore, an accurate Wiener filter can be constructed without estimation of noise power spectrum. In the later part of this paper, we will verify the performance of Anscombe transformation by numerical calculation and show the good accordance between two distributions.

3) Temporal K-L transformation By performing the temporal K-L transformation on the Anscombe transformed data sequence, the noise reduction task is further simplified from a 4D problem to a 3D process.

If we view the time-activity curve for pixel j as a realization of a random sequence, the dynamic image sequence can be viewed as an ensemble of M realizations of this random procedure [12, 13]. The temporal K-L basic vectors then can be obtained as the eigenvectors of the covariance matrix K_t ($K\times K$) of the time behavior of the image sequence, i.e., they are the row of A defined by

$$K_t A^T = A^T D \tag{3}$$

where $D = diag\{d_1,\ldots,d_K\}$ and d_i is the i-th eigenvalue of K_t.

The K-L transformation of the whole dynamic data along its time axis can be represented by a matrix A_M of the following form [12]

$$A_M = A\otimes I_M = \begin{bmatrix} \begin{bmatrix} a_{11} & & 0 \\ & \ddots & \\ 0 & & a_{11} \end{bmatrix} & \begin{bmatrix} a_{12} & & 0 \\ & \ddots & \\ 0 & & a_{12} \end{bmatrix} & \cdots & \begin{bmatrix} a_{1K} & & 0 \\ & \ddots & \\ 0 & & a_{1K} \end{bmatrix} \\ \begin{bmatrix} a_{21} & & 0 \\ & \ddots & \\ 0 & & a_{21} \end{bmatrix} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \begin{bmatrix} a_{K1} & & 0 \\ & \ddots & \\ 0 & & a_{K1} \end{bmatrix} & \cdots & \cdots & \begin{bmatrix} a_{KK} & & 0 \\ & \ddots & \\ 0 & & a_{KK} \end{bmatrix} \end{bmatrix}, \tag{4}$$

in which A_{ij} are the elements of the matrix A, I_M denotes the $M\times M$ identity matrix and $\otimes$ represents the Kronecker product.

Apply A_M to observed dynamic data $\boldsymbol{g}$, the temporal K-L transformation and its inverse one may then be defined as

K-L transform: $\boldsymbol{y} = A_M \boldsymbol{g}'$ (5)

Inverse K-L transform: $\boldsymbol{g} = A_M{}^T \boldsymbol{y}$. (6)

That means, all pixels in the dynamic sequence are subject to the same transformation. The covariance matrix of dynamic data along the time axis can be estimated from [5, 13]

$$K_{t;kl} = \tfrac{1}{M-1}\sum_{i=1}^{M}(g'_{k,i}-\overline{g}_k)(g'_{l,i}-\overline{g}_l) \tag{7}$$

where

$$\overline{g}_k = \tfrac{1}{M}\sum_{i=1}^{M} g'_{k,i}\,, \quad k,l = 1,\ldots,K \tag{8}$$

B. Spatio-Temporal Wiener Filter for Gated SPECT

In this section, we formulate the spatio-temporal Wiener filter in the K-L domain after Anscombe transformation. Corresponding to Eq.(2), the MMSE or Wiener restoration is produced by [12]

$$\boldsymbol{p}' = K_{p'}(K_{p'}+K_{n'})^{-1}\,\boldsymbol{g}' \tag{9}$$

where K_p and K_n are the covariance matrices of the dynamic image and noise after Anscombe transformation, respectively.

To simplify the overall time-space restoration problem, here we assume that K_p, the covariance matrix of both spatial and temporal dimensions, is separable into purely spatial and temporal components as follows:

$$K_{p'} = K_t \otimes K_s\,. \tag{10}$$

The validation of the separability assumption has been discussed in [13]. Usually, it can be applied directly to imaging of motion-free objects. Here we assume it is true because the motion information can be reflected from temporal fluctuations of the signal [12]. This assumption is one strong basis for the following models of this paper.

By applying the temporal K-L transformation on Eq.(2), we have that

$$\begin{aligned} \boldsymbol{y} = A_M\boldsymbol{g}' &= A_M\boldsymbol{p}' + A_M\boldsymbol{n}' \\ &= \boldsymbol{y}_1 + \boldsymbol{n}_1 \end{aligned} \tag{11}$$

where we have defined

$$\boldsymbol{y}_1 = A_M\boldsymbol{p}', \quad \text{and} \quad \boldsymbol{n}_1 = A_M\boldsymbol{n}'\,. \tag{12}$$

We now choose to perform a Wiener filter restoration upon the transformed data as seen in Eq.(11). The Wiener restoration can be written as

$$\boldsymbol{y}_1 = K_{y1}(K_{y1}+K_{n1})^{-1}\,\boldsymbol{y} \tag{13}$$

where K_{n1} is the covariance matrix of the transformed noise and K_{y1} is the covariance of the transformed image data $\boldsymbol{y}_1$. To calculate $\boldsymbol{y}_1$, we have

$$\begin{aligned} K_{y1} &= \mathrm{E}[\boldsymbol{y}_1\boldsymbol{y}_1{}^T] \\ &= \mathrm{E}[A_M\boldsymbol{p}'\boldsymbol{p}'^T A_M{}^T] = A_M K_p A_M{}^T \end{aligned}$$

$$=(A \otimes I_M)(K_t \otimes K_s)(A \otimes I_M)^T$$
$$= AK_t A^T \otimes Ks \ . \quad (14)$$

According to the definition of the temporal K-L matrix A in Eq.(3), we have $AK_t A^T = D$, and this yields

$$K_{y1} = D \otimes K_s \quad (15)$$

which is block diagonal in the K-L domain.

Similarly, we obtain the covariance matrix of noise n_1

$$K_{n1} = \mathrm{E}[\boldsymbol{n}_1 \boldsymbol{n}_1^T]$$
$$= \mathrm{E}[A_M \boldsymbol{n}' \boldsymbol{n}'^T A_M^T] = A_M K_n A_M^T . \quad (16)$$

As we described above, after Anscrombe transform, the Poisson distributed data become approximately Guassian distributed with the variance equal to 0.25. Since noise is uncorrelated with the image and also uncorrelated with each other, K_n can be simply expressed as $s_n^2(I_M \otimes I_K)$, where s_n^2 is a constant equal to 0.25. By substituting K_n in Eq.(16), we have

$$K_{n1} = (A \otimes I_M)(s_n^2 I_M \otimes I_K)(A \otimes I_M)^T$$
$$= s_n^2 (A A^T \otimes I_M) = s_n^2 I_K \otimes I_M , \quad (17)$$

which demonstrates that the noise property of constant variance remains for all components in the K-L domain.

Examining Eq.(13), we see that all the indicated matrices are block diagonal. Thus, the Wiener filter of Eq.(17) can be expressed as K independent filters in each of the blocks, as shown below

$$\boldsymbol{y}_{1i} = d_i K_s (d_i K_s + s_n^2 I_M)^{-1} \boldsymbol{y}_i \quad , i = 1, 2, \ldots, K \quad (18)$$

If we assume spatial stationarity for the spatial correlations, then Ks is a block-Toeplits matrix and can be approximated by a circulant matrix [12]. Then all calculations in Eq.(18) can be performed by Fourier computations. The final form of our filter in the K-L domain is

$$M(w_s, w_z, k_q) = \frac{S_{y_i}(w_s, w_z, k_q) - s_n^{\ 2}}{S_{y_i}(w_s, w_z, k_q)} \quad (19)$$

where S_{yi} is the 3D discrete Fourier transformation (FFT) of data $\boldsymbol{y}_i$ at the point (?,z,?). The availability of the noise covariance matrix in this method eliminates the need for the estimation or separation difficulty of noise from signal.

Furthermore, the SNR (signal-to-noise ratio) for each component [14] in the K-L domain is:

$$SNR = \frac{K_{y1}}{K_{n1}} = \frac{K_y}{K_{n1}} - 1 \quad (20)$$

Since after Anscombe transformation, $K_{n'} = 0.25I$, by rewriting Eq.(20), we have:

$$SNR + 1 = 4D \otimes K_s . \quad (21)$$

Eq.(21) means that the SNR of K-L domain sinogram components can be reflected by the eigenvalues of the temporal covariance matrix. This provides one measure of the significance of the components in the K-L domain. In other words, performing the temporal K-L transformation makes each K-L domain component independent and the SNR of each component decrease monotonically for the lower eigenvalues.

In summary, the spatio-temporal filtering task is greatly simplified by first applying the Anscombe transformation to all the sinogram data, which converts Poisson distributed noise into Gaussian distributed one with constant variance. Then by performing the temporal K-L transformation on the Anscombe transformed data sequence, the task is further simplified from a 4D problem to a 3D spatial process. In the K-L domain, the noise property of constant variance remains for all components, while the signal-to-noise ratio decreases monotonically for lower eigenvalue components. An accurate Wiener filter is then constructed for each component of a 3D data set.

III. gMCAT SIMULATION STUDY

A. Creation of Projection Data

The major goal of our simulation is to evaluate the performance of the proposed multi-frame filtering algorithm and compare it with that of other methods such as Shepp-Logan filter, Hann filter and single-frame 3D Wiener filter.

We used the gated mathematical cardiac torso (gMCAT) phantom to simulate 16-interval gated activity projection data without attenuation. The relative activity levels of the heart: lungs: liver: kidney: spleen: sternum were 1.0: 0.03: 0.69: 0.84: 0.96: 0.12, respectively, simulating the distribution of Tc^{99m} [7]. Noise-free, 128x128 emission projections over 128 views spanning 360° were simulated. Poisson noise, at a level of total counts per time frame over all angles, was added to these emission projections to simulate conditions observed in clinical application.

B. Filtering Implementation

The implementation procedure we proposed for noise treatment of gated SPECT data is summarized as follows:

- Apply Anscombe transformation to all the observed dynamic projection data.
- Construct an estimate of the matrix K_t according to Eq.(11), then calculate K-L transform matrix A from K_t.
- Perform temporal K-L transformation on all data obtained by Eq.(1) with Eq.(5).
- Calculate S_{yi} for each frame and then perform 3D Wiener filtering according to Eq.(19) frame-by-frame in the Fourier domain.
- Perform inverse K-L transformation on the filtered data by A_M^T.
- Inversely Anscombe transform all the filtered projection data using p'^2-1/8.
- Normalize smoothed projection data.

For comparison purpose, the projection data were reconstructed by conventional FBP method after different kinds of filtering, such as Shepp-Logan filter, Hanning filter, single-frame Wiener filter, and this proposed spatio-temporal Wiener filter. The cutoff frequencies for Hann and Shepp-Logan filters are 0.35 and 0.25, respectively.

IV. Results and Discussion

A. Experimental Verification of Anscombe Transformation

To verify the performance of Anscombe transformation, we performed a numerical calculation. For given mean 0<?<300, we generated Poisson probability function $f(x)$. By transforming each x to y by $y=(x+3/8)^{1/2}$ and assigning probability function $f(x)$ to its corresponding $f(y)$, we compared transformed mean and variance with the approximated $(l+1/8)^{1/2}$ and 0.25, respectively, as shown in Figure 1. It can be found that, except for ?<5, the difference between the transformed mean and the approximated $(l+1/8)^{1/2}$, as well as the difference between the transformed variance and 0.25, are both less than 10^{-3}. This verifies that a Poisson variable can be converted to a Gaussian variable with a know variance 0.25, as proven by Anscombe [10, 11]

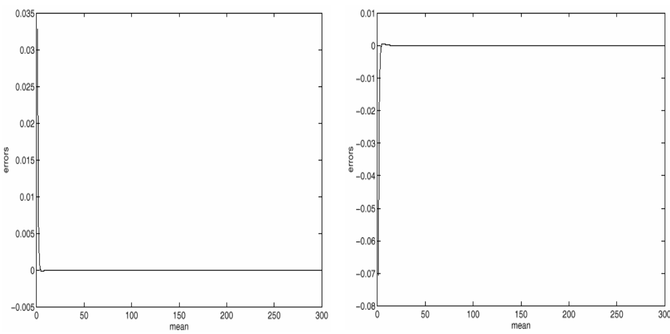

Figure 1: Left: the error of the Anscombe transformed mean and the approximated mean. Right: the error of the transformed variance and 0.25.

Strictly speaking, the transformation from Poisson distribution to Guassian distribution can not be achieved exactly since one is the discrete random process, while the other is continuous random process. The claim that this transform can achieve the goal is only based on the first two orders of statistics.

B. Gated Cardiac SPECT Simulation Studies

The reconstructed transverse slices obtained using different filtering approaches, as shown in Figure 2 as well as the activity profiles along the lines as indicated shown in Figure 3, indicate that the spatio-temporal Wiener filter outperforms the conventional frame-by-frame spatial restoration method. In Figure 2, the images come from the same reconstructed slice obtained in different scans. For all filtering methodologies considered, single-frame 3D Wiener provides better noise suppression than other conventional spatial filters, although a little over-smoothed, while our method further improves the noisy appearance of image sequence without sacrifice of resolution. It reveals that the spatio-temporal Wiener filter does a better job in terms of filtering the Poisson noise in the degraded image sequence over spatial filtering.

Application of the proposed transformation to the gMCAT phantom data yields the component images of Figure 4. The superior performance of this transform in ordering the K-L components by image quality is apparent. It can be seen even in the lower engenvalue components, such as in frame 13, there still has some signal information, visible from noise.

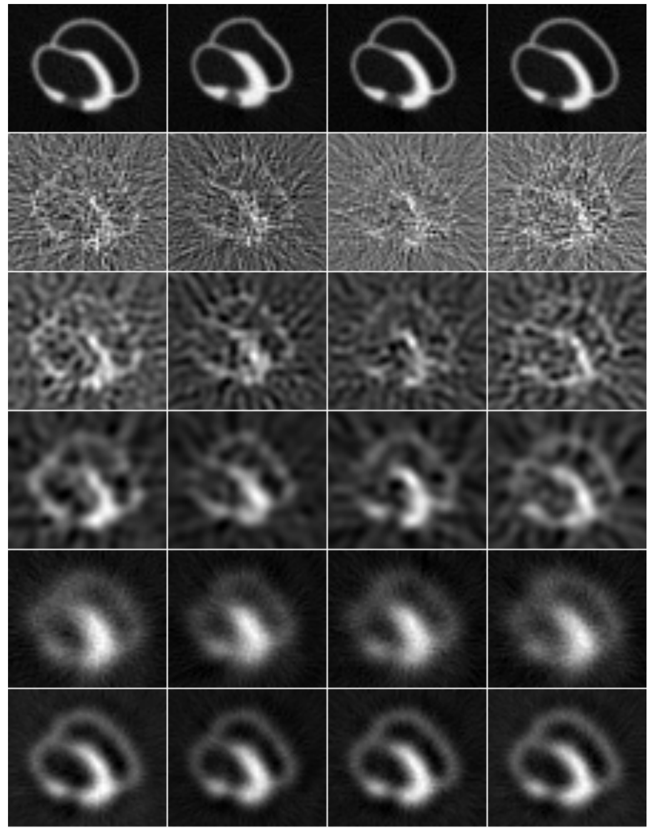

Figure 2: Comparison of transverse MCAT slices reconstructed using different filtering approaches. From left to right: frame 1, 5, 9, and 13. From top to bottom: noise-free, noisy data filtered by ramp, Shepp-Logan, Hann, single-frame 3D Wiener filter and proposed method.

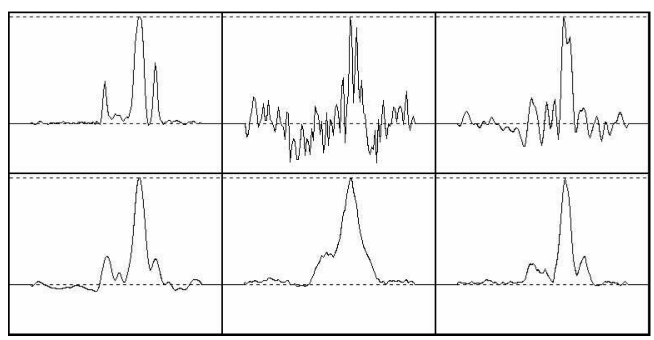

Figure 3: Comparison of the activity profiles drawn from slice 61 reconstructed using different filtering methods along the lines indicated. Top, from left to right: noise-free, noisy data filtered by ramp, Shepp-Logan. Bottom, from left to right: Hann, single-frame 3D Wiener filter and proposed method.

V. Conclusions

The method described here considers the characteristics of Poisson noise as well as spatio-temporal correlation of gated cardiac SPECT projection data. It utilizes Anscombe transformation to construct more accurate Wiener filters and K-L transformation to translate the spatio-temporal filtering of the entire image sequence into a spatial frame-by-frame filtering in K-L domain. Since all temporal and spatial correlations are utilized, it is an optimal solution in contrast to other sub-optimal ones that utilize independent component estimation. By this approach, the spatio-temporal filtering can also be achieved at a reasonable computational cost. The computer simulations are very encouraging, by visual judgement, as compared to those conventional frame-by-frame spatial filtering.

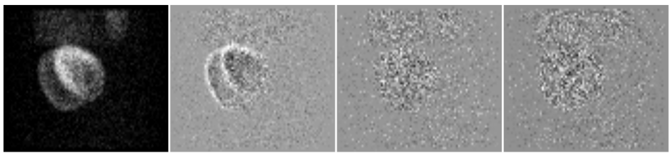

Figure 4: K-L domain component images (sinograms, slice 104) in different frames. From left to right: frame 1, 2, 3, and 13.

As we mentioned before, the proposed method maximizes the SNR in each successive transform components, in a same manner as the K-L transform maximizes the data variance in successive components. Therefore, in situations where the covariance matrix has only a few significant eigenvalues, only the corresponding K-L components need to be filtered and restored, resulting in further reduction of the computational burden. Furthermore, to improve the performance of the filtering, a weighting window could be applied to make the filter adaptive to each component SNR. The weighting for the areas of lower SNRs should be lessened, while areas of higher SNRs should be enhanced, in order to preserve edge information and reduce noise. There are many methods to design an adequate weighting window [15]. Considering the relationship between the component SNRs and the eigenvalues, the weighting can be selected to be a function of the eigenvalues, which is currently under investigation.

To fully exploit the noise reduction capabilities of a spatio-temporal filter without introducing artifacts into the sequence, the filter could be adaptive both spatially and temporally. Such a filter, however, can be very costly in terms of computational requirement [2, 16]. Our method, which can be an alternative approach towards that end, is theoretically based and has shown very encouraging results by simulation studies. Further validation by phantom experiments and clinical data is needed.

V. References

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