Noise Properties of Low-Dose CT Projections and Noise Treatment by Scale Transformations

Hongbing Lu, *Member, IEEE*, Ing-Tsung Hsiao, *Member, IEEE*, Xiang Li, and Zhengrong Liang, *Member, IEEE*

***Abstract*---Projection data acquired for image reconstruction of low-dose computed tomography (CT) are degraded by many factors. These factors complicate noise analysis on the projection data and render a very challenging task for noise reduction. In this study, we first investigate the noise property of the projection data by analyzing a repeatedly acquired experimental phantom data set, in which the phantom was scanned 900 times at a fixed projection angle. The statistical analysis shows that the noise can be regarded as normally distributed with a nonlinear signal-dependent variance. Based on this observation, we then utilize scale transformations to modulate the projection data so that the data variance can be stabilized to be signal independent. By analyzing the relationship between the data standard deviation and the data mean level, we propose a segmented logarithmic transform for the stabilization of the non-stationary noise. After the scale transformations, the noise variance becomes approximately a constant. A two-dimensional Wiener filter is then designed for an analytical treatment of the noise. Experimental results show that the proposed method has a better noise reduction performance without circular artifacts, by visual judgment, as compared to conventional filters, such as the Hanning filter.**[·]

I. Introduction

Projection data acquired for image reconstruction of low-dose computed tomography (CT) are degraded by many factors, including Poisson noise, logarithmic transformation of scaled measurements, and pre-reconstruction corrections for system calibration [1]. All these factors complicate noise analysis on the projection data and render a very challenging task for noise reduction in order to maintain the high image quality of currently available CT technologies. Up to now, various forms of filtering techniques have been developed to spatially smooth the projection data and/or the reconstructed CT images [1-3]. One approach models the data noise by Gaussian distribution with variance proportionally depending on the signal or density of the data [1]. It utilizes a nonlinear anisotropic diffusion filter to smooth the data noise. Another approach employs an adaptive trimmed mean filter to reduce streak artifacts, which are resulted from excessive X-ray photon noise in low-dose CT projections [2]. Although both of them succeed in some degrees for noise reduction prior to image reconstruction, the assumption of the noise model is not justified in their applications and further development is then limited. Sauer and Liu [3] developed a non-stationary filtering method for the anisotropic artifacts in the image reconstruction. Although it utilizes local noise properties to construct a set of non-stationary filters, the method is a post-processing type approach on the images. This type of filtering usually gains noise reduction at the cost of resolution.

As a method for statistically treating any non-normally distributed, signal dependent noise, scale transformations are widely used to stabilize the variance of empirical variables or measurements [4-6]. The purpose of the transformations is to modulate the measurements so that the modified data variance is stable and further signal independent. That means, if the variance of a variable tends to change with the mean, the variance could be stabilized by a suitable scale transform so that the derivation of a theoretically based approach would be applicable for more accurate noise treatment. For example, if the variance of the low-dose CT projection data is proportional to the mean, a square-root type transformation may be considered. If the variance changes quadratically with the mean value, a logarithmic transform may be a better choice to stable the data [5]. Since scale transform-based approaches have the advantages of efficiency in computation and uniqueness in determining the solution, they have been extensively utilized in nuclear medicine field. For example, Pélégrini et al [7] and Lu et al [8] have used a modified square-root transformation (also referred to as Anscombe transform [4]) to convert Poisson noise into normally distributed one with constant variance. This transformation greatly simplifies the treatment of Poisson noise for emission computed tomography, such as SPECT (single photon emission computed tomography) and PET (positron emission tomography). In their non-parametric regression sinogram smoothing approach, La Riviere et al [9] also showed that the choice of a square-root link function in the penalized objective function would lead to reconstructed images with satisfactory uniform and isotropic resolution. In order to extend the well-studied maximum-likelihood (ML) expectation-maximization (EM) algorithm to a general reconstruction problem (such as preprocessing of sinograms), Nuyts et al [10] employed noise equivalent counts (NEC) --- scaling and shifting

---

[·]This work was supported in part by the National Heart, Lung and Blood Institute under Grant No. HL54166, National Institute of Neurological Disorder and Stroke under Grant No. NS33853, and Established Investigator Award of the American Heart Association. The authors greatly acknowledge Dr. Jiang Hsieh, who is with the Applied Science Laboratory of GE Medical Systems, Milwaukee, WI 53201, USA, (telephone: 262-785-5824, e-mail: jiang.hsieh@med.ge.com), for providing the phantom projection data of this study.

H. Lu, I-T Hsiao, and X. Li are with the Radiology Department, State University of New York, Stony Brook, NY 11794, USA, (telephone: 631-444-7921, e-mail: {hblu}, {ihsiao}, {xli}@clio.rad.sunysb.edu).

Z. Liang is with the Departments of Radiology, Computer Science and Biomedical Engineering, State University of New York, Stony Brook, NY 11794, USA, (telephone: 631-444-7837, e-mail: jzl@clio.rad.sunysb.edu).

0-7803-7324-3/02/$17.00 (C) 2002 IEEE

transformations --- to convert arbitrary sinogram noise into one that is approximately Poisson distributed.

In this study, we first investigated the nonlinear noise property of the low-dose CT projection data (after pre-reconstruction corrections and/or calibrations) by analyzing a repeatedly acquired experimental data set from a phantom at a fixed projection angle. The statistical analysis showed that the noisy projection data can be regarded as normally distributed with a nonlinear signal-dependent variance. Based on this observation, we utilized scale transformations to modulate the measurements so that the variance of the projection data would be stabilized to satisfy the requirement for a theoretically-based approach. After the transformation, the noise variance became approximately a constant and a two-dimensional (2D) Wiener filter was then designed for an analytical treatment of the non-stationary noise. Several scale transformations were investigated in order to find an appropriate and optimal one for the noisy data.

II. Methods

A. Analysis of Noise Properties

To analyze the noise property of low dose CT data, we first repeatedly acquired projection measurements of a physical phantom at a fixed angle for 900 times by a GE spiral CT scanner. The measurements were calibrated and corrected as projection data to satisfy the Radon transform. The probability distribution of the calibrated data from channel 600 is shown on the left of Figure 1 with comparison to corresponding Poisson, Gamma, and Gaussian probability distribution functions (PDF). It can be clearly seen that the noise distribution of the projection data has an approximated Gaussian functional, instead of usually assumed Poisson distribution. The mean distribution along detector channels is shown on the right of Figure 1. It reflects the line integrals at a projection angle across a cylinder phantom. To make the analysis of the noise property more complete, the phantom was designed to make the projection acquired cover the possible data range as much as possible.

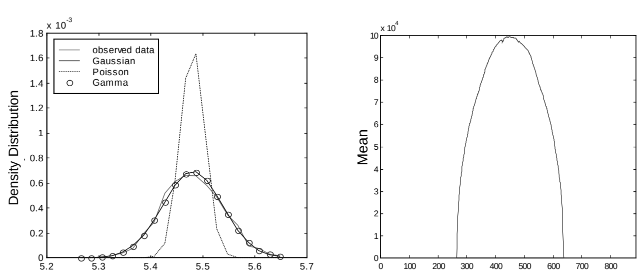

Figure 1. Noise property of acquired low-dose CT projection data. The number of channels per view is 888 and the number of measurements is 900. Left: PDFs of channel 600. Right: Mean distribution of the data across the channels.

The relationship between the standard deviation and the mean of the calibrated data for all the channels is shown on the left of Figure 2, while the relationship between noise variance and data mean is shown on the right of Figure 2. It is seen that the relationship between the data standard deviation (or noise variance) and the data mean is nonlinear and may not have an analytical functional formula.

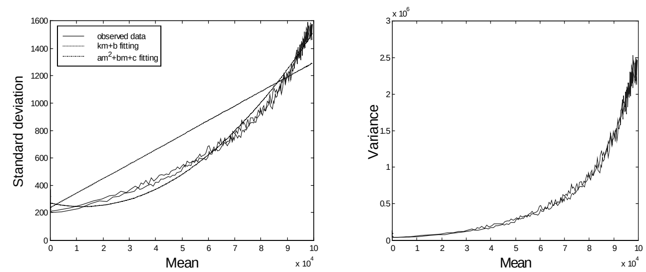

Figure 2. Left shows the standard deviation--mean curve. Right shows the variance--mean curve.

B. Scale Transformations

The general purpose of scale transformations is to change the scale of the measurements in order to make the statistical analysis more valid, especially for the analysis of variance. As we know, for the measurements with constant variance, enormous smoothing approaches have been proposed for accurate treatment of the noise. However, for those with non-stationary variance, conventional low-pass filters would not be appropriate any more because of their spatially-invariant assumption and ignoring for signal-dependent nature of the noise. As an alternative, iterative methods consider the noise properties accurately, but their iterative nature demands a heavy computational effort and furthermore their associated cost-function regularization and iterative convergence have been remained as research topics for years [11]. If we can convert the non-stationary noise of low-dose CT projection data into approximately stationary one with accurate treatment of the noise, the conventional filtered backprojection (FBP) reconstruction would provide the similar improvement as the iterative methods without both the computing burden and the regularization and convergence issues. In the following, we will present a brief review on scale transformations and their implementations.

If the dependence of the variance on the mean level has a mathematical form of

$$\sigma_x^2 = f(m), \quad (1)$$

where σ_x^2 is the variance of variable x with mean equal to m, then for any function $g(x)$ we have approximately [6]

$$\sigma^2(g(x)) \approx f(m)(dg(m)/dm)^2, \quad (2)$$

where $\sigma^2(g(x))$ is the variance of transformed data $g(x)$, and $dg(m)/dm$ reflects the first derivative of function g. Then if we want $\sigma^2(g(x))$ to be a constant, C^2 say, we must have

$$g(m) = \int \frac{C\, dm}{\sqrt{f(m)}}. \quad (3)$$

For example, if the variance σ_x^2 tends to be proportional to the mean level, then the Anscombe transform, as we applied before on the treatment of Poisson noise in SPECT image [8], would be a choice. If the standard deviation σ_x tends to be proportional to the mean level *m*, there is *g(m)* proportional to *log(m)*, then the logarithmic scaling would be a choice. In reference [5], Bartlett gave a summary of scale transformations that have been used on empirical, statistical data with general distributions. Of course, a constant variance is not the only conditions that we seek. Another important condition of whether the transformed variable is normally distributed shall be considered when using variance analysis and scale transformation. It also should be mentioned that the approximation of a scaled variable to a normal distribution is only based on the first and second moments of the distribution.

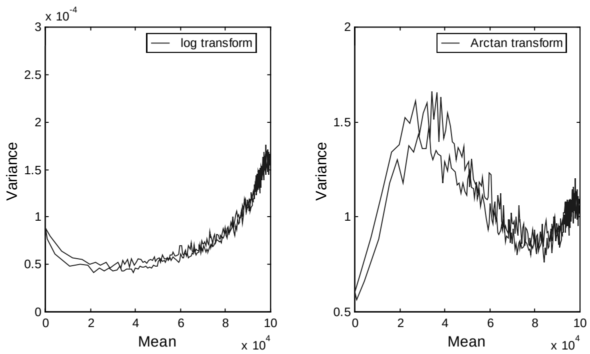

Figure 3. Left --- Variance-mean curve for log transform Eq.(4). Right --- Variance-mean curve for Arctan transform Eq.(5).

To find an appropriate scale transform for data with arbitrary distribution, the first step is to get an expression of *f(m)*. According to the relationship between the standard deviation and the mean of the CT projections, we first use $\sigma_x = km+b$ and $\sigma_x = am^2+bm+c$ to fit the curve, as shown on the left of Figure 2, with the minimum least-square criterion. The quadratic fitting looks better that the linear fitting. The corresponding transform *g(x)* for the linear fitting $f(m) = (km+b)^2$ is the logarithmic transform, as shown below

$$g(x) = \frac{C}{k}\ln(kx+b) + C_1 \qquad (4)$$

where *C* is the expected constant standard deviation for transformed data and C_1 is an arbitrary constant. The scale transformation corresponding to the quadratic fitting $f(m) = (am^2+bm+c)^2$ is the inverse tangent transform like

$$g(x) = \frac{2C}{\sqrt{4ac-b^2}}\, arctg\, \frac{2ax+b}{\sqrt{4ac-b^2}} + C_2 \qquad (5)$$

where C_2 also is an arbitrary constant. After applying these scale transforms to the calibrated projection data respectively, the variance-mean curves of transformed data are shown in Figure 3. It can be seen that though the range of variation of the data variance is reduced dramatically, from $0\sim2.6x10^6$ to $0.4\sim1.8$, it is still far from being a stable variable, especially for that of the inverse tangent transform. It's not surprising to see when we tried to utilize Wiener filter to smooth the logarithmic transformed data, ring artifacts appeared near the rotation center in the reconstructed images, as shown in Figure 6. The reason for the appearance of artifacts can be explained as the non-stability of the variance, especially around low mean level and high mean level region. Considering the unmatched parts of the linear fitting with the standard deviation-mean curve, we then proposed to use a segmented logarithmic transform to improve the variance property of the transformed data, as shown on the left of Figure 4. The original standard deviation-mean curve was fitted into three parts by three fitting lines, under the minimum least-square criterion. Thus the corresponding logarithmic transform consists of three different log transforms applied to the data at different mean levels, which we call as segmented logarithmic transforms. The variance-mean curves for segment-transformed data at different mean levels are shown in Figure 5. It can be clearly seen that the variance of transformed data is greatly stabilized and approximately equal to 1, especially at both low-level and high level mean regions. The improvement provided by the proposed segmented logarithmic transforms is significant, which can be seen in Figure 6.

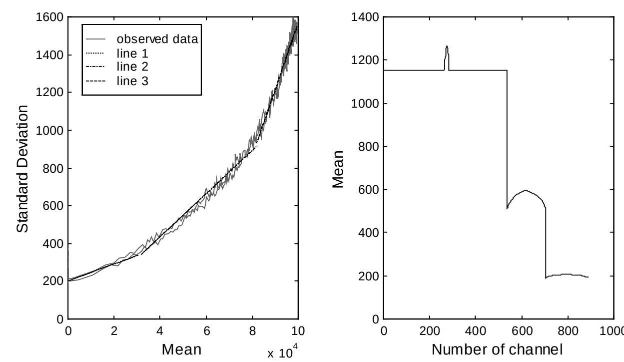

Figure 4. Left --- Standard deviation-mean curve and three fitting lines. Right --- Mean distribution of segmented transformed data.

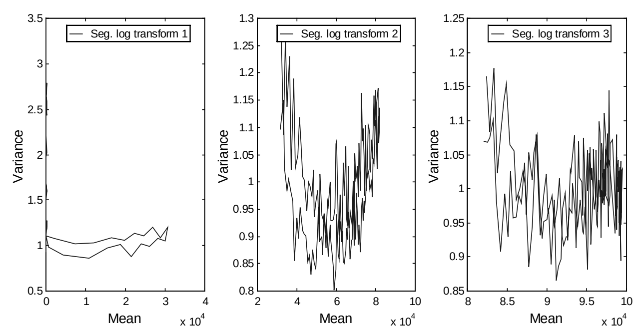

Figure 5. From left to right --- Variance-mean curves for three data regions scaled by different log transforms.

C. Wiener Filtering

After the scale transformations, the modulated projection data achieved approximately a constant variance. Then an accurate filtering method can be derived. Here the Wiener filter is an optimal choice. If we assume that each sinogram is spatially stationary, then the Wiener filter for 2D sinogram in frequency domain is expressed as

$$H(\omega_s, k_\theta) = \frac{S_g(\omega_s, k_\theta) - \sigma_n^{\ 2}}{S_g(\omega_s, k_\theta)} \qquad (6)$$

where S_g is the 2D discrete Fourier transform (FT) of scaled sinogram $\boldsymbol{g}$ and (ω_s, k_θ) denotes the 2D FT coordinates. Notation σ_n^2 is the noise variance. Since the variance of scaled projection data is approximately constant after the scale transformations, it eliminates the difficulty of the estimation of noise power spectrum in the Wiener filtering. That means, σ_n^2 can be regarded as a constant, which can be derived from *f(m)* and *g(m)*, during filtering process.

It could be argued that many other types of filters might have better properties in terms of noise suppression, edge preservation, and etc, than the Wiener filter. But, the objective of this paper is mainly to demonstrate the utilization of scale transformations on noise analysis and filtering, not to investigate an optimal filter for this low-dose CT application. Wiener filter is selected based on its well-known properties of minimum least-square error on a linear system, its easy implementation and its efficiency in computation.

III. ENXPERIMENTAL RESULTS

Experimental phantom projections were acquired by the same GE spiral CT scanner with a fan-beam curved detector array. The phantom used here is a cylinder with several small details, as shown in the top-left of Figure 4. The number of channels per view is 888 with 984 views evenly spanned on a circular orbit of 360°. The detector array is on an arc concentric to the X-ray source with a distance between the X-ray source and the rotation center equal to 541 mm. The distance from the rotation center to the curved detector is 408.075 mm. The detector cell spacing is 1.0239 mm. After pre-reconstruction correction for system calibration, the calibrated projection data were filtered by different kinds of smoothing methods for comparison purpose. The filtered projection data were then reconstructed by the conventional FBP method with the Ramp filter. To see the influence of the variance stability of the scale transformations on sinogram smoothing, the projection data were filtered by the Wiener filter after different kind of the scale transformations.

The reconstructed slices by different filtering approaches for the same phantom slice respectively are shown in Figure 6. The "noise-free" data are the average of 19 repeatedly acquired datasets, while the "noisy data" is a single dataset. The noise level is quite high. The reconstructed images filtered by the Hanning filter reveal the tradeoff between image resolution and noise. Insufficient noise suppression was observed for higher cutoff frequencies. For lower cutoff frequencies, image resolution is sacrificed. Although the single logarithmic transform provides some degree of noise smoothing, it generates circular artifacts around the center of the rotation in the reconstruction images (bottom left of Figure 6). This is due to its poor ability to stabilize the data variance, as shown on the left of Figure 3. The scaled variance after the single log transform fluctuates between 0.4~1.8. The variance fluctuation is reduced to the range of 0.8~1.3 after applying the segmented log transforms. The variance-mean curves of the segmented log transforms are more stabilized, especially at the regions with low and high mean levels (on the left and right of Figure 5). The reconstructed images in Figure 6 also show that the segmented log transforms can eliminate the circular artifacts and provide a better noise suppression than the low-pass filters, such as the Hanning filter, without the sacrifice of the spatial resolution.

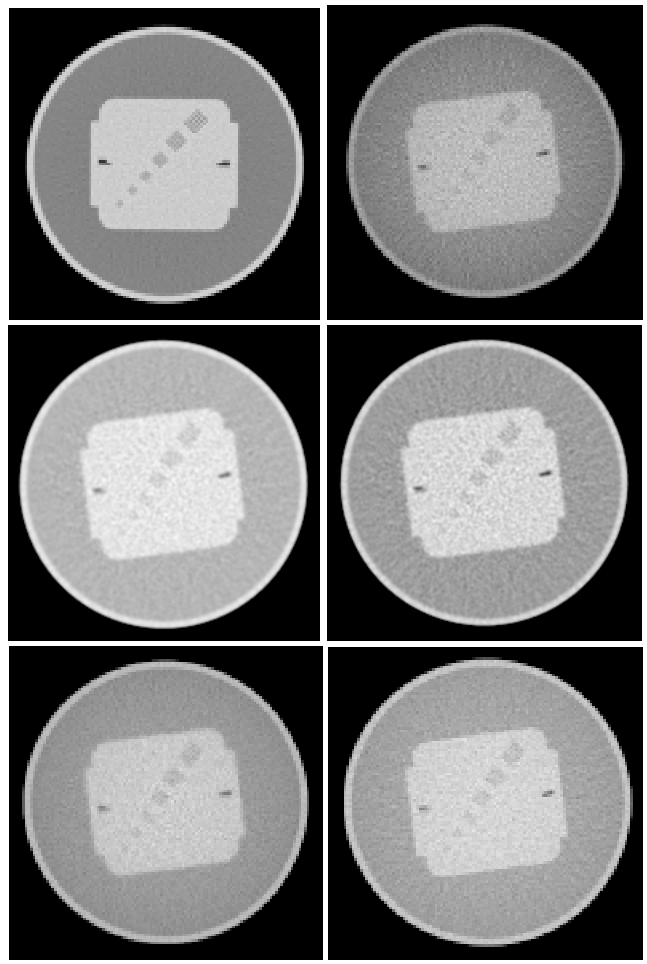

Figure 6. Comparison of reconstructed CT images using different filtering approaches. Top: left --- noise-free; and right --- noisy. Middle: left --- Hanning filter (cutoff frequency $f_c = 0.25$); and right --- Hanning filter (f_c=0.40). Bottom: left --- log transform + Wiener filter; and right --- segmented log transform + Wiener filter.

IV. CNONCLUSIONS

In this study, we investigated the non-stationary noise properties of low-dose CT projection data and presented a method to transform the non-stationary noise into a much more stabilized one for the purpose of facilitating the treatment of the noise. The method depends on the prior knowledge of the noise properties. In the low-dose CT problem, the noise properties are known by phantom studies. This method is an attempt to capture the benefits of statistical modeling in a non-iterative manner. More accurate treatment of non-stationary noise can be achieved by the proposed technology by using other analytical means, rather than

0-7803-7324-3/02/$17.00 (C) 2002 IEEE

Wiener filter, while maintaining the computational efficiency. Considering the Gaussian functional and the mean-variance relation of the low-dose CT projection data, the penalized weighted least-square (PWLS) smoothing approach would be a choice for an alternative optimal solution, which is currently under investigation [12].

V. References

[1] O. Demirkaya, "Reduction of noise and image artifacts in computed tomography by nonlinear filtration of the projection images", *Proc. SPIE Med. Imaging*, vol. 4322, pp. 917-923, 2001.

[2] J. Hsieh, "Adaptive streak artifacts reduction in computed tomography resulting from excessive x-ray photon noise", *Med. Physics*, vol. 25, no.11, pp. 2139-47, 1998.

[3] K. Sauer and B. Liu, "Non-stationary filtering of transmission tomograms in high photon counting noise", *IEEE Trans. Med. Imaging*, vol. 10, no.3, pp. 445-452, 1991.

[4] F.J. Anscombe, "The transformation of Poisson, binomial and negative-binomial data", *Biometrika*, vol. 35, pp. 246-254, 1948.

[5] M.S. Bartlett, "The use of transformations", *Biometrics*, vol. 3, no.1, pp. 39-53, 1947.

[6] W.J. Curtiss, "On transformations used in analysis of variance", Annals of mathematical Statistics, vol. 14, pp. 10, 1943.

[7] M. Pélégrini, H. Benali, G. El Fakhri, and et al, "Two-dimensional statistical model for regularized backprojection in SPECT", *Phys. Med. Biology*, vol. 43, pp. 421-434, 1998.

[8] H. Lu, Z. Liang, D. Chen, and et al, "A combined transformation of ordering SPECT sinograms for signal extraction from measurements with Poisson noise", *Proc. SPIE Med. Imaging*, vol. 4322, pp. 943-951, 2001.

[9] P.J. La Rivière and Xiaochuan Pan, "Nonparametric regression sinogram smoothing using a roughness-penalized Poisson likelihood objective function", *IEEE Trans. Med. Imaging*, vol. 19, pp. 773-786, 2000.

[10] J. Nuyts, C. Michel, and P. Dupont, "Maximum-likelihood expectation-maximization reconstruction of sinograms with arbitrary noise distribution using NEC-transforms", *IEEE Trans. Med. Imaging*, vol. 20, no.5, pp.365-375, 2001.

[11] J.A. Fessler and W.L. Rogers, "Spatial resolution properties of penalized likelihood image reconstruction: space invariant tomographs", *IEEE Trans. Image Proc.*, vol. 5, pp. 1346-1358, 1996.

[12] H. Lu, X. Li, J. Hsieh, and et al, "Analytical treatment of non-stationary noise of low-dose CT projection data by penalized weighted least-square smoothing in the K-L domain", *Proc. SPIE Med. Imaging*, 2002 (submitted).