# Spatially-Adaptive Analytical Reconstruction of Quantitative Gated Cardiac SPECT in KL Domain

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***Abstract* -- Gated cardiac SPECT imaging has been widely used in the evaluation of cardiac functions. The acquired four-dimensional (4D) sinogram data have both inter-frame correlation among the time sequence and intra-frame correlation within each 3D frame. The unique features of Karhune-Loève (KL) transform for de-correlating the data sequence for noise reduction, feature extraction, etc, have been well recognized in the past. Based on the previous work utilizing the KL features, we propose an improved method for 4D quantitative SPECT reconstruction for both noise reduction and computation saving. It considers the similarity among the gated frames in a cardiac cycle. This similarity exists when the heart goes from diastole to systole and then back to diastolic state. All the dynamic sequences are first grouped by unsupervised clustering means based on their similarity, where the similarity between any two frames is measured inversely by the summation of the intensity errors between all sinogram pixels of the two frames. Those frames sharing similar motion state are grouped together until all the error summation exceeds a preset threshold. Then different KL transforms are applied independently in different groups to de-correlate the inter-frame correlation. In the KL domain, an analytical algorithm based on Novikov's inverse formula for non-uniform attenuation compensation is developed to reconstruct each principal component for its corresponding image component in the KL domain. Noise reduction is adaptive to each principal component in the proposed algorithm. The higher order principal components could be discarded for further noise reduction and computation saving. The 4D image sequence is reconstructed by the corresponding inverse KL transform of each group, respectively. Both simulation study using the NCAT phantom and clinical experiment using the DigiRad SPECT system were performed, showing that the proposed method can achieve better performance than the one using a global KL transform for all frames.**

## I. Introduction

SPECT (single photon emission computed tomography) is a cost-effective, widely-used diagnostic modality which utilizes radiotracers to directly label tissue functions. Gated SPECT imaging studies yield useful kinetic information and thus has plays an important role in clinical treatment. The acquired four-dimensional (4D) sinogram data have both inter-frame correlation among the time sequence and intra-frame correlation within each 3D frame. Currently, the time sequence is usually reconstructed frame-by-frame using a conventional filtered back-projection (FBP) method, which ignores the temporal correlation and lacks quantitative capability. The quantitative aspect can be improved by the ordered-subsets expectation-maximization (OSEM) iterative reconstruction frame-by-frame, but the inter-frame correlation is still not utilized. Recent studies have been devoted to consider the whole 4D sinagram as an indivisible one. By applying the Karhune-Loève (KL) transform to address the temporal correlation, Kao, *et al.*[1] proposed a pre-reconstruction temporal sinogram smoothing for dynamic PET (positron emission tomography) imaging by discarding the higher order principal components, which were dominated by noise (i.e., larger variances). Wernick, *et al.*[2] employed the KL transform to seek a penalized weighted least-squares (PWLS) estimate of the entire sequence in the KL domain for a fast reconstruction of dynamic PET. Narayanan, *et al.*[3,4] extended the KL-based framework [1,2] to gated SPECT and showed that the framework was almost equally effective as the 4D iterative pML reconstruction but much faster in computation. Our study shifted the above iterative approaches to an analytical one for gated cardiac SPECT and achieved satisfactory performance [5-7].

A limitation of the above approaches is that they are based on the assumption that all pixels obey the same time activity curve (TAC), thus a pooled estimation of covariance matrix for KL transform can be used. In fact, the gated cardiac sinagram data are usually acquired in a cardiac cycle. The difference between frames in systolic and diastolic periods would be significantly different. Even two neighboring frames may show distinct spatial variety. The reconstruction based on a global KL transform would be degraded by such activity variation.

In this paper, we present a potential refinement of the previous methods. Those frames sharing similar TAC activities are subject to a corresponding KL transform, instead of all frames are subject to the same KL transform.

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## II. METHODS

In our previous method [5-7], KL transform was performed along the time axis to de-correlate the gated SPECT images. Analytical approach based on Novikov's inverse formula [8] is applied in KL domain to compensate for non-uniform attenuation and obtain quantitative reconstruction. This type of reconstruction could be very promising for gated cardiac SPECT study because it makes use of the correlation among images sequences and converts the 4D reconstruction problem into 3D operations. However, this method may not accurately capture the features of the time behavior due to a single KL strategy.

In this paper, we proposed an improved version of above approach by following steps below: 1) group frames in projection space based on their similarity of temporal behavior, 2) apply different KL transforms to different groups, 3) reconstruct image components in each group to obtain quantitative results, and 4) apply inverse KL transform respectively in each group. In the following sections, more details on the implementation of each step are given.

### *A. Grouping Method:*

In the first step, the time-activity behavior between different gated frames could be reflected by the similarity of sinograms in the projection domain. This similarity could be calculated by unsupervised clustering method, and the grouping method consists of three steps below.

*Step1:* Subtract the intensity of each pair of pixels locating at the same position of two different frames, and then calculate the summation of the absolute errors to get a distance matrix between any two frames, as shown in table I.

Table I

Distance Matrix

| | $p_k(1)$ | ... | $p_k(j)$ | ... |
|---|---|---|---|---|
| $p_k(1)$ | $d_k(1,1)$ | - | - | - |
| ... | ... | ... | ... | ... |
| $p_k(i)$ | $d_k(i,1)$ | ... | $d_k(i,j)$ | ... |
| ... | ... | ... | ... | ... |

where *d(i,j)* represents the similarity between the i-*th* and j-*th* frames of projection sequences. Suffix *k* denotes the k-*th* iteration. It is noted that the initial size of distance matrix should be *K*K*, where *K* is the total number of frames. Moreover, *d(i,j)* should be equal to *d(j,i)* and thus the distance matrix is a triangle matrix .

*Step 2:* Find out the minimum value in the matrix and its index, i.e., *d(m.n)*, and then combine $p_k(m)$ and $p_k(n)$ into one group $p_{k+1}(q)$, where $q = \{m.n\}$ . Update the table by modifying the distance between group *p(q)* and other group *p(l)* according to:

$$d_{k+1}(l,q) = \min(d_k(l,m), d_k(l,n)) \tag{1}$$

*Step 3:* Repeat step 2 until certain condition are satisfied, as mentioned below. After the grouping process terminates, the frames with indexes in one group are classified to have similar time behavior and will be treated together.

Ideally, the iteration should be terminated when the grouped frames in each group have a similar TAC activity and an obvious difference could be observed between different groups. However, it is difficult to set a criterion describing this condition exactly. By careful observation, we classify the frames of the whole cardiac circle into three groups representing systolic, diastolic and transitional state, respectively. By this definition, the criterion for terminating the iterative process could be clearly described as follows: terminate the iteration when the newly updated matrix has only three groups and further iteration would reduce the number of groups.

### *B. KL Transformation:*

In this section, presentation is made in two dimensions for simplicity. In each group, assume the vector of measured time-activity curve at pixel (*i,j*) in sinogram space is represented by :

$$\lambda_{i,j,h} = [\lambda_{i,j}^1, \lambda_{i,j}^2, \cdots, \lambda_{i,j}^{g_h}]^T \tag{2}$$

where $g_h$ is the number of frames within the h-*th* group of the cardiac cycle. In our study, $h = 1,2,3$ and $\sum_{h=1}^{3} g_h = K$ .

Element (*k,l*) of the time covariance matrix $P_h$ in the h-*th* group can be estimated by:

$$[P_h]_{k,l} = \frac{1}{I*J-1} \sum_{i,j} \left(\lambda_{i,j}^k - \overline{\lambda}^k\right)\left(\lambda_{i,j}^l - \overline{\lambda}^l\right) \tag{3}$$

where *I*J* is the total number of pixels in one frame and $\overline{\lambda}^k$ represents the estimated mean value of frame *k* by:

$$\overline{\lambda}^k = \frac{1}{I*J} \sum_{i,j} \lambda_{i,j}^k \tag{4}$$

By singular value decomposition, the eigenvectors $M_h$ of $P^h$ can be obtained from:

$$P_h^{time} M_h^T = M_h^T \cdot V_h \tag{5}$$

where $V_h = diag\{v_1, v_2, \cdots, v_k\}$ and $v_k$ denotes the *k*-th eigenvalue of $P^h$ .

In each group, by multiplying the time-activity of pixels one-by-one with $M_h$, the temporal *KL* transform of the dynamic data can be performed by:

$$A_h = M_h \cdot \lambda_{i,j,h} \tag{6}$$

Where $A_h$ is a $g_h \times 1$ vector and could be presented as $A_h = \left[a_{i,j}^1, a_{i,j}^2, \cdots a_{i,j}^k, \cdots, a_{i,j}^{g_h}\right]^T$. $a_{i,j}^k$ is the *k*-th *KL* domain element at pixel (*i,j*). The KL transformed sequence then can be obtained by organizing $A_h$ for the corresponding pixels (*i,j*) in the KL domain. Since the KL transform is applied along the time dimension and the mean of each frame is computed from all *I*J* pixels in that frame using equation (4), all the *I*J* pixels are subjected to the same transformation.

*C. Analytical Quantitative Reconstruction in KL Domain:*

To perform the quantitative image reconstruction in analytical way, we have modified the Novikov's explicit inverse formula [8] to make it applicable in the KL domain.

Let (*x,y*) be the stationary coordinate in image domain and $(t,\theta)$ be the rotation coordinate in sinogram space. The KL domain Novikov inverse formula can be expressed as:

$$\phi(\vec{r}) = \frac{1}{4\pi} div \int_0^{2\pi} \vec{j}[\exp\left([D\mu]_\theta(s,t)\right)\widetilde{q}(t,\theta)]\Big|_{\substack{s=\vec{r}\bullet\vec{j} \\ t=\vec{r}\bullet\vec{k}}} d\theta \tag{7}$$

where $\vec{j} = (\cos\theta, \sin\theta)$, $\vec{k} = (-\sin\theta, \cos\theta)$, ***div*** is the divergence operation, $\phi(\vec{r})$ is the reconstructed image frame from its corresponding sinogram frame $A(t,\theta)$ in the KL domain and

$$\widetilde{q}(t,\theta) = e^{-h_1}\{\cos(h_2)\widetilde{q}_1(t,\theta) + \sin(h_2)\widetilde{q}_2(t,\theta)\} \tag{8}$$

$$\widetilde{q}_1(t,\theta) = H\cos(h_2)e^{h_1}A(t,\theta) \tag{9}$$

$$\widetilde{q}_2(t,\theta) = H\sin(h_2)e^{h_1}A(t,\theta) \tag{10}$$

with $h_1 = \frac{1}{2}[R\mu](t,\theta)$, $h_2 = [Hh_1](t,\theta)$. The operators *H*, *D*, and *R* represent the Hilbert transform, the divergent beam transform, and the Radon transform, respectively, and are defined as follows:

$$[Hg](s) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{g(\tau)}{s-\tau}d\tau \tag{11}$$

$$[D\mu]_\theta(s,t) = \int_t^\infty \mu_\theta(s,\tau)d\tau \tag{12}$$

$$[R\mu](s,\theta) = \int_{-\infty}^{\infty}\mu_\theta(s,t)dt \tag{13}$$

In each group, the above *KL* domain quantitative reconstruction is performed frame-by-frame for each principal component. For each reconstructed pixel $\phi_{m,n}^l$ in the *KL* domain, the obtained result is presented as: $\Phi_{m,n,h} = (\phi_{m,n,h}^1, \phi_{m,n,h}^2, \cdots \phi_{m,n,h}^l, \phi_{m,n,h}^{l+1}, \cdots, \phi_{m,n,h}^{g_h})$. Since the higher-order components with smaller eigenvalues may have little information, only the first *l* low-order components may be reconstructed, i.e., $\Phi_{m,n}' = (\phi_{m,n}^1, \phi_{m,n}^2, \cdots \phi_{m,n}^l)$ ($l \leq g_h$), for further noise reduction and computing efficiency. After the Novikov inversion in the KL domain, an inverse *KL* transform for the reconstructed components is applied to obtain the dynamic images in the original space for each group.

$$\hat{f}_{m,n,h} = M_h^T \Phi_h \tag{14}$$

## III. Results

To evaluate proposed method, we reconstructed the gNCAT [9] phantom with computer simulation, as well as clinical data from patients acquired by a DigiRad SPECT.

Based on the rule of thumb from statistics know as "scree method" [10], the minimum number of components to be retained was chosen based on the inflection point or the point at which the eigenvalue spectrum began to flatten before further decreasing. Fig. 1 shows an example of normalized eigenvalue spectrum for the gNCAT phantom. Based on the rule, there are 9 components are reconstructed in 3 groups for total frames.

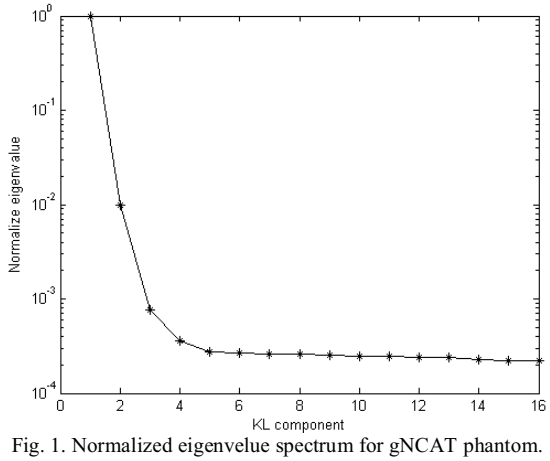

Fig. 1. Normalized eigenvelue spectrum for gNCAT phantom.

Fig.2 and Fig. 3 demonstrate reconstructed results using different methods for gNCAT phantom and clinical data. The results treated with global KL transform strategy are also given for comparison. The marked regions in Fig.2 show the improvements with proposed method. Some details that were lost using previous method are clearly captured when using proposed method.

## IV. Conclusion

In this paper, a refinement of our previous analytical reconstruction method for quantitative gated cardiac SPECT sequences has been presented. The method is based on the observation that there are strong signal correlations among different frames of the 4D SPECT sequences, especially among those frames which share similar TAC activities.

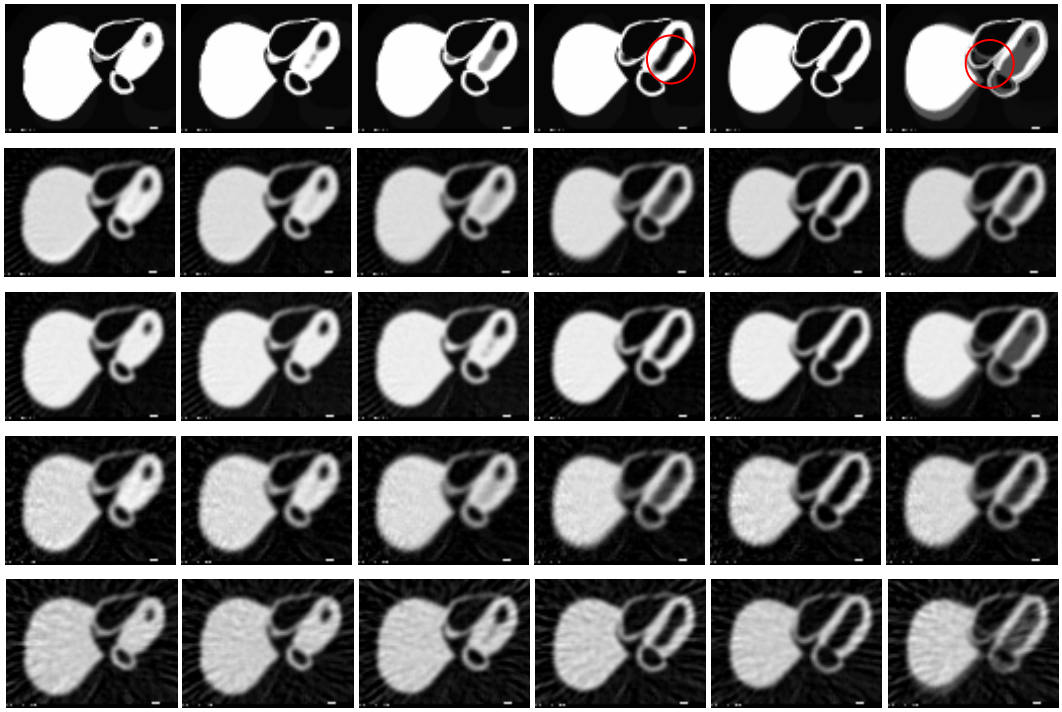

Fig. 2. Reconstructed images of gated NCAT phantom using different reconstruction methods. From left to right: 1st, 4th, 7th, 11th, 14th and 16th frame of gated sequence. From top to bottom: Original distribution, reconstruction by the global KL transform strategy in noise-free situation, reconstruction by proposed grouping method in noise-free situation, reconstruction by the global KL transform strategy in noisy situation, reconstruction by proposed grouping method in noisy situation.

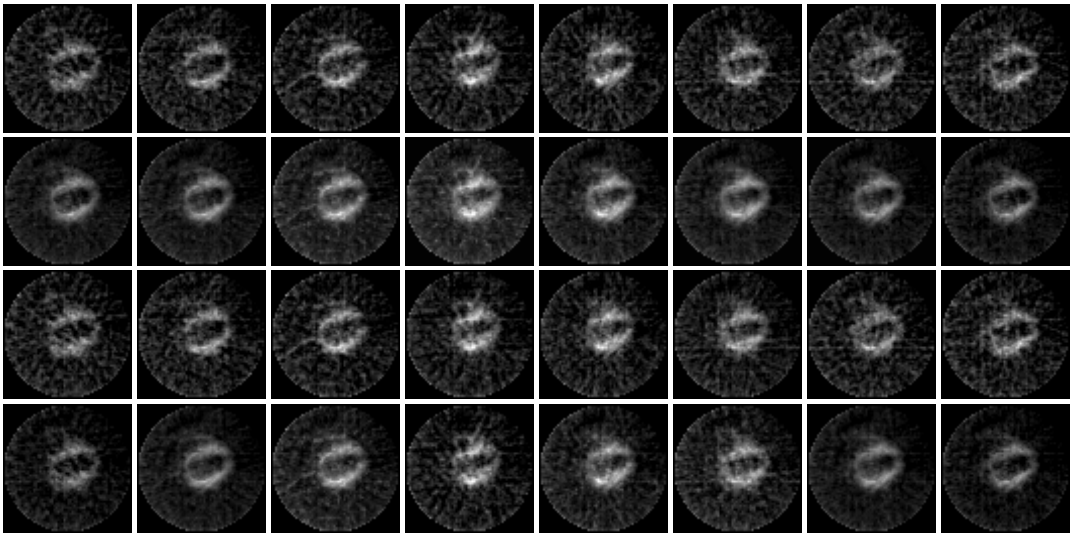

Fig. 3. Reconstructed images of clinical data using different reconstruction methods. From left to right: eight gated frames. From top to bottom: reconstruction by FBP, by the global KL transform strategy, by the clustering method proposed by Wernick *et al*., by proposed grouping method.

Definition and classification of the cardiac motion in gated sequences could enhance the use of correlation by improving the reconstruction performance. Compared with the previous study, the grouping strategy proposed in this paper could greatly overcome the shortage of one KL strategy and thus enhance the correlation of gated sequences. The KL transform provides a unique way to de-correlate the information and rearrange data according to their eigenvalues. The analytical algorithm is based on the Novikov's inverse formula and can compensate for the non-uniform attenuation, thus it provides quantitative results. Furthermore, for KL components associate with smaller eigenvalues, which are mainly dominated by noise, could be discarded for noise reduction and computation saving. Since the covariance matrices are estimated from those frames sharing similar TAC behavior, the proposed method would capture more detailed motion characteristics compared with the previous one.

Different from the previous one applying the KL transform

to all gated sequences in a cardiac cycle, the presented method employs spatially-adaptive strategy. Unsupervised clustering is used to identify frames with similar time behavior. Since the proposed method is automatic, easy to use, and has less negative effects, it has the potential for 4D gated cardiac SPECT reconstruction.

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